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Three-dimensional magnetotelluric modeling in anisotropic media using edge-based finite element method

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ABSTRACT

It is important to understand how magnetotelluric (MT) modeling can most effectively be performed in general anisotropic media. However, previous studies in this area have mainly focused on the use of one-dimensional (1D) and two-dimensional (2D) algorithms. Thus, building on earlier work, it is important to study the performance of three-dimensional (3D) modeling in arbitrary conductivity media; therefore, an edge-based finite element (FE) method has been developed for 3D MT modeling in arbitrary conductivity media. This approach is based on the initial derivation of a series of equivalent variational equations that are based on Maxwell equations, generated using the weighted residual method. Specific values were then obtained for coefficient matrixes of this edge-based FE method using hexahedral meshes, and the algorithm was verified by comparing its results with finite difference (FD) solutions generated using a 2D anisotropic model. Finally, the results of a 3D anisotropic model were analyzed detailed for three conditions; another 3D anisotropic model was designed and its results were compared with two isotropic models'.

1. Introduction

The magnetotelluric (MT) method is an important geophysical method that has been widely used in many fields such as mineral resources survey, exploration of oil & gas and the investigation of deep Earth electrical structures. However, the interpretation of MT data generally assumes an isotropic medium, and numerous studies have shown that the Earth is anisotropic (Christensen, 1984; Wannamaker, 2005). A general ignorance about the influence of anisotropic media on MT has thus likely led to misinterpretations, which means that it is both important and meaningful to study the modeling and inversion of this approach in anisotropic media. Although approaches for the modeling and inversion of MT in one-dimensional (1D) and the modeling in two-dimensional (2D) anisotropic media are relatively mature, inversions in 2D and three-dimensional (3D) anisotropic media and the modeling in 3D anisotropic situations urgently need to be developed. A number of analytical solutions are available for 1D situations on the basis of numerous studies (O'Brien and Morrison, 1967; Reddy and Rankin, 1971; Dekker and Hastie, 1980; Yin, 2000; Pek and Santos, 2002), while the electrical and magnetic field cannot easily be separated in 2D cases. Nevertheless, a large number of studies have been carried out in this area (Heise and Pous, 2001, 2003; Yin, 2003; Hu et al., 2013; Huo et al., 2015); Pek and Verner (1997), for instance, developed a staggered-grid finite difference (FD) method for application in arbitrary 2D anisotropic media, which has had significant influence in the field. In later work, Li (2002) developed a modeling approach using the finite element (FD) method in 2D generally anisotropic media; the results of this research are in close agreement with solutions based on the FD method. Li and Pek (2008) subsequently developed an adaptive FE modeling algorithm in 2D general anisotropic media. In 3D anisotropic media, there are some studies of Marine Controlled-source Electromagnetic (Yin et al., 2014; Cai, et al., 2015). To date, however, just a handful of studies have applied MT modeling to 3D anisotropic media; one early example was the work of Martrinelli and Osella (1997) who presented a Rayleigh-Fourier method which allows for vertical anisotropy, while Weidelt (1999) later developed a staggered-grid FD method for use in 3D general anisotropic conductivity media that does not require a significant computational increase. Li (2000) presented a detailed nodal-based FE method for MT modeling, generating variational equations using the Galerkin weighted residual method, while Wang and Fang (2001) developed an FD algorithm for multicomponent electromagnetics in 3D anisotropic formations but did not present a case study example. Häuserer and Junge (2011) simulated a 3D anisotropic anomaly based on real data from Uganda, while Löwer and Junge (2017) studied the spatial and frequency-dependent behavior of phase tensors and tipper vectors using the FD method within an anomalous 3D anisotropic conductive body.

There are several serious problems inherent to the use of node-based elements within an FE model (Jin, 2002). Thus, a 3D MT

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numerical modeling algorithm was implemented using an edge-based element method in arbitrary conductive media. The system of equations derived was a large, sparse matrix equation. It was solved using the bi-conjugate gradient-stabilized (Bi-CGSTAB) method combined with the symmetric successive over-relaxation (SSOR) preconditioner. The modeling method was validated comparing its calculation results with those obtained using FD method (Pek and Verner, 1997) for a 2D anisotropic model. The results of a 3D anisotropic model, a 3D anisotropic anomaly embedded in an isotropic half-space, were analyzed detailed for three conditions and some conclusions were obtained. At last, another 3D anisotropic combined model and two isotropic models were designed, the resistivities of the two special 3D isotropic model are the same as the resistivities of the anisotropic model in the x- and y-direction respectively. The results of this anisotropic model were compared with these two isotropic models'.

2. The modeling of MT in 3D anisotropic media

The study space utilized in this work is shown in Fig. 1, divided into air zone and subterranean zone. The sources are located on the top surface ABCD.

2.1. Differential equations

In the case of a quasi-stationary approximation, we consider a harmonic time dependence $e^{-i\omega t}$, and ignore displacement currents for MT. Thus, Maxwell's equations are changed, as follows:

$$\nabla \times \mathbf{E} = i\omega\mu\mathbf{H} \tag{2.1}$$

$$\nabla \times \mathbf{H} = \tilde{\sigma}\mathbf{E} \tag{2.2}$$

$$\nabla \cdot \mathbf{H} = 0 \tag{2.3}$$

$$\nabla \cdot \mathbf{E} = 0 \tag{2.4}$$

where **E** and **H** are the electric field and the magnetic field respectively, ω is the angular frequency, μ is the magnetic permeability of the media (considered in this case to be equal to the value in a vacuum, μ_0), and $\tilde{\sigma}$ is a tensor conductivity in anisotropic medium, as follows:

$$\tilde{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \tag{2.5}$$

Two methods (Yin, 2000; Pek and Santos, 2002) are available to define the conductivity tensor, the latter of which is adopted in this paper.

The magnetic field **H** can be obtained from Eq. (2.1). Substituting it into Eq. (2.2) generates Eq. (2.6), as follows:

$$\nabla \times \nabla \times \mathbf{E} - i\omega\mu\tilde{\sigma}\mathbf{E} = 0 \tag{2.6}$$

2.2. Variational problem

We applied the Galerkin variant of the weighted residuals method (Xu, 1994) to generate the variational equation. This was done by first multiplying Eq. (2.6) by the variation in the electric field, $\delta\mathbf{E}$, and then integrating across the whole study space. At the same time, we utilized vector identity (Eq. (2.7)) and the divergence theorem (Eq. (2.8)):

$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = (\nabla \times \mathbf{A}) \cdot \mathbf{B} - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \tag{2.7}$$

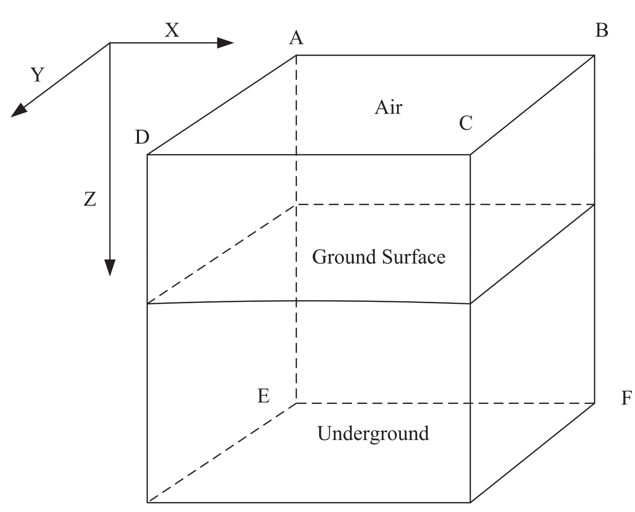

Fig. 1. The study space utilized for this research.