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Experimental demonstration of nonlinear quantum metrology with optimal quantum state

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Nonlinear quantum metrology may exhibit better precision scalings. For example, the uncertainty of an estimated phase may scale as $\Delta\phi \propto 1/N^2$ under quadratic phase accumulation, which is $1/N$ times smaller than the linear counterpart, where N is probe number. Here, we experimentally demonstrate the nonlinear quantum metrology by using a spin-I ($I > 1/2$) nuclear magnetic resonance (NMR) ensemble that can be mapped into a system of $N = 2I$ spin-1/2 particles and the quadratic interaction can be utilized for the quadratic phase accumulation. Our experimental results show that the phase uncertainty can scale as $\Delta\phi \propto 1/(N^2-1)$ by optimizing the input states, when N is an odd number. In addition, the interferometric measurement with quadratic interaction provides a new way for estimating the quadrupolar coupling strength in an NMR system. Our system may be further extended to exotic nonlinear quantum metrology with higher order many-body interactions.

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Xinfang Nie and Jiahao Huang contributed equally to this work.

I. INTRODUCTION

Quantum metrology is the science of pursuing high-precision parameter measurements by utilizing the principles of quantum mechanics [1, 2]. Quantum metrology has been widely applied in many areas, such as magnetic/electric field sensing [3, 4], measurement of weak signals [5], quantum computation [6], gravitational wave detection [7–10] and so on. In general, the unknown parameter χ is encoded in a Hamiltonian $\mathcal{H} = \chi\hat{A}$, where $\hat{A}$ is a generator. By preparing an input state and letting it evolve under $\mathcal{H}$, a relative phase $\phi(\chi)$ is accumulated and χ can be extracted via interfering. For some parameters such as the transition frequency between two atomic energy levels or the Larmor frequency in an nuclear magnetic resonance (NMR) system, the phase accumulation is linear. The measurement precision of relative phase ϕ may be enhanced from $\Delta\phi \propto N^{-1/2}$ to $\Delta\phi \propto N^{-1}$ by using an N-particle entangled states [11–16]. The quantum metrology under linear phase accumulation have been demonstrated in a large number of experiments [17–23].

Nonlinear quantum metrology [24–28] opens new frontiers such as different measurement precision scaling [29–35]. The nonlinear quantum metrology can be achieved by s-wave scattering [35], Kerr nonlinearity [36, 37], quadrupolar coupling [38, 39] and dipole polarizability [40] etc., where the encoded phase accumulation becomes nonlinear (i.e., the output-input transformation under $\mathcal{H}$ is nonlinear [41]). Theoretical studies on nonlinear quantum metrology show that, the measurement uncertainty may be at best scale as $\Delta\phi \propto N^{-k}$ with a k-body interaction phase accumulation ($k > 1$), which is different from the linear quantum metrology [29, 32].

It is well known that, the phase measurement precision is limited by the quantum Cramer-Rao bound, $\Delta\phi \geq 1/\sqrt{F_Q}$, where $F_Q = 4\Delta^2\hat{A}$ is the quantum Fisher information (QFI) for pure input states [2, 42]. Thus, the standard deviation of the generator $\Delta\hat{A}$ determines the lower bound of the phase uncertainty. Generally, the standard deviation of the generator in linear quantum metrology is proportional to the probe number $\Delta\hat{A} \propto N$, while the standard deviation of the generator in nonlinear quantum metrology may be a power dependence on the

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