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# Upper bound analysis of differential velocity sideways extrusion process for curved profiles using a fan-shaped flow line model

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An analytical model for predicting the shapes of rectangular bars with variable curvatures along their lengths through a novel forming method, differential velocity sideways extrusion (DVSE), previously proposed by the authors, has been developed on the basis of the upper bound method. A new flow line function was presented to describe its deformation field. The plastic deformation zone (PDZ) was assumed to be fan-shaped, where the trajectory of the material flow within the PDZ had an elliptic shape. The proposed continuous flow line function was validated using finite element simulations. The flow patterns, extrusion pressure, curvature, and effective strain predicted by the analytical solutions agreed well with modelling results. Compared to the classical discontinuous simple shear model of channel angular extrusion (CAE) with a 90° die, the new approach was shown to predict the effective strain more closely.



## 1. Introduction

The demand for using extruded aluminium profiles as structural components on aircraft, trains and cars has been increasing nowadays due to lightweight design, where a reduced consumption of fuel and therefore a decreased emission of $CO_2$ can be achieved. In automobiles, aerospace, and shipbuilding industry, curved profiles are largely used for the manufacturing of ultra-light complex structures with high stiffness and strength due to aerodynamics, structural properties, and design reasons [1–4].

Curved profiles are mostly achieved by conventional bending procedures such as stretch bending, press bending, rotary draw bending and roll bending. However, most of them have disadvantages such as cross-section deformations and springback of profiles during the bending process which need to be avoided through expensive tools [5–10], thus inevitably significantly increasing the manufacturing costs. Some novel stress superposed cold bending techniques, i.e. torque superposed spatial (TSS) bending and superposed three-roll-bending with subsequent profile deflection, have been proposed to improve the forming limitations [11–14]. It was found that cross-sectional deformations and springback of curved profiles can be greatly reduced because of the superposition of torsion or compression with the external bending moment.

Recently, several novel extrusion-bending integrated methods have been developed. One is curved profile extrusion (CPE) proposed by Kleiner and co-workers [15,16] to decrease the manufacturing procedures of curved profiles. During CPE the metal billets are directly formed into curved profiles within only one extrusion procedure, thus significantly improving the manufacturing efficiency. This method is based on the conventional straight extrusion process, where a bending device is directly installed behind the die exit orifice to deflect the extruded profile so that it comes out of the die with the prescribed curvature. Muller [17,18] used a segmented regulating guiding device which is composed of serially placed bending discs at the die exit, to bend the extruded profile. Another way of extruding curved profiles is by exploiting an inclined die to adjust the material flow velocity distribution over the profile cross-section. Shiraishi and co-workers [19–21] developed a novel extrusion-bending integrated forming process for producing curved bars and tubes, in which a plasticine billet is extruded through a die aperture inclined towards the central axis of the container at a predetermined angle. It was found that the curvature of the extruded bars and tubes can be varied by adjusting the inclination angle of the die aperture, i.e., a greater inclination angle results in a greater curvature.

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**Nomenclature**

| | |
|---|---|
| $D_1$ | Width of the billet (mm) |
| $D_2$ | Width of the extruded profile (mm) |
| $dh, dl$ | Height and length of the differential element of the plastic deformation zone (PDZ) (mm) |
| $dV$ | Volume of the differential element of the PDZ ($mm^3$) |
| $h$ | Height of the dead metal zone (DMZ) (mm) |
| $k_0, k_f$ | Initial and final shear yield stresses of the material (MPa) |
| $\overline{k}$ | Mean shear yield stress of the material (MPa) |
| $l$ | Die bearing length (mm) |
| $l_1, l_2$ | Transient billet lengths with velocity $v_1, v_2$ respectively (mm) |
| $m$ | Constant friction factor |
| $P_1, P_2$ | Extrusion pressures of the upper and lower punches (MPa) |
| $P_{1u}, P_{2u}$ | Upper bound of extrusion pressures of the upper and lower punches (MPa) |
| $R_c$ | Bending radius of the profile (mm) |
| $S_0$ | Cross-sectional area of the billet ($mm^2$) |
| $S_3, S_4$ | Cross-sectional areas of the related profile ($mm^2$) |
| $S_f, S_v$ | Areas of frictional and velocity discontinuity surfaces ($mm^2$) |
| $\Delta v$ | Amount of velocity discontinuity (mm/s) |
| $v_1, v_2$ | Extrusion velocities of the upper and lower punches (mm/s) |
| $v_{1e}, v_{2e}$ | Maximum and minimum material flow velocities across the die exit orifice (mm/s) |
| $v_3, v_4$ | Velocities at the volume (mass) centre of the related profile (mm/s) |
| $v_p$ | Velocity of particle p moving on the curve MN (mm/s) |
| $w$ | Thickness of the billet and extruded profile (mm) |
| $\dot{W}_{def}$ | Power dissipated in the plastic deformation zone (N·mm/s) |
| $\dot{W}_e, \dot{W}_i$ | External and internal power supplies (N·mm/s) |
| $\dot{W}_{S_f}, \dot{W}_{S_v}$ | Power dissipated on the frictional and velocity discontinuity surfaces (N·mm/s) |
| $\overline{y}_3, \overline{y}_4$ | Coordinates of the volume (mass) centre of the related profile (mm) |

*Greek symbols*

| | |
|---|---|
| $\theta$ | Angular position along the flow line (°) |
| $\overline{\varepsilon}$ | Effective strain |
| $\dot{\overline{\varepsilon}}, \dot{\overline{\varepsilon}}_m$ | Effective strain rate, mean value of the effective strain rate of the PDZ ($s^{-1}$) |
| $\kappa$ | Bending curvature of the extruded profile ($mm^{-1}$) |
| $\lambda$ | Extrusion ratio |
| $\xi$ | Eccentricity ratio |

The authors proposed a novel extrusion-bending process, named differential velocity sideways extrusion (DVSE), and its feasibility was studied [22,23]. The basic principle of this method is bending profiles simultaneously while extruding, which is achieved by controlling the gradient of the internal material flow velocity at the die exit orifice owing to the difference of relative moving velocities of two extrusion punches. It has been experimentally shown that by adjusting the extrusion velocity ratio of the two opposed punches as well as the extrusion ratio, curved profiles with adjustable curvatures can be obtained in one procedure. The quantitative relationship between curvature of the extrudate and the process parameters (the punch velocity ratio, extrusion ratio etc.) need to be developed for guiding the tooling design and forming process. Generally this can be achieved by theoretical analysis and numerical simulation such as the finite element (FE) modelling. The upper bound theorem has been extensively used to predict the forming pressure and analyse the deformation characteristics in the extrusion of profiles, ring rolling and forging processes, and to determine and minimise the exit profile curvature in the extrusion process of non-symmetrical profiles [24–28], etc. An approximate analytical model for predicting the forming pressure and curvature of the extrudate (round bar) has been developed by the authors based on the upper bound method and the rigid block model, where the plastic deformation zone (PDZ) was considered as consisting of several single shear planes and the modes of deformation were composed of rigid blocks of material separated by the velocity discontinuity planes [29]. The simple shear model is normally used in the upper bound analysis for approximation especially when the geometry is complex, e.g., 3D round bars in Ref [29]. In practice, material flow should be continuous without velocity discontinuity. On the other hand, FE modelling has also been widely used in the metal forming process since it can provide an accurate visual description of the material flow. However, one simulation case of the DVSE process requires huge computational resources. It is not efficient to design the DVSE process by FE modelling directly since numerous computations at different process conditions are needed. Instead, analytical expression can be achieved by upper bound theorem, which can be more easily applied in the process design and optimisation.

Therefore, in this paper, an upper-bound model, based on a more precise and realistic fan-shaped flow line model in which any part of the material will face a gradual and continuous change in its velocity rather than abrupt variations (velocity discontinuity) throughout the deformation process, is proposed for estimating the distribution of the dead metal zone (DMZ), extrusion pressure, curvature and effective strain of the extruded profile during the novel DVSE process. To focus on the flow field in the PDZ of the DVSE process, the geometry factor in the previous work [29] is simplified here by studying a rectangular bar, where the thicknesses of the die entrance and exit channels are the same in the configuration of the DVSE process. A finite element model has been developed in parallel with the analytical model to assess the accuracy and validity of the model. The effects of extrusion velocity ratio and extrusion ratio were investigated in detail. The new approach is also compared to the classical discontinuous simple shear model of channel angular extrusion (CAE) with a 90° die. The findings provide a deeper understanding needed for wider application of the DVSE forming technique.

## 2. Theoretical model

### 2.1. Upper bound model

The present analytical method is formulated on the basis of the upper bound theorem for a rectangular material undergoing plane strain extrusion, i.e. strain along thickness direction (normal to the paper) is assumed to be zero. This happens when the thicknesses of the die entrance and exit channels are the same in the configurations of the DVSE process. Similar assumption has been made previously by Kwan and co-workers [30], in which they argued that the process of equal-cross-section lateral extrusion may be assumed as a plane-strain problem, even if the cross-section is

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