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Modelling and effective properties prediction of metal foams

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Abstract

This work focuses on finding methodologies to describe the effective elastic properties of metal foams. For this purpose, numerical methods and analytical models, were used. Kelvin cells and Weaire–Phelan structures were modelled to represent both open and closed-cell representative unit-cells. These unit-cells were then subjected to different homogenization methods: (i) Far field methods with single freedom constraints, where it was used two different approaches based on the load case. (ii) Asymptotic Expansion Homogenization (AEH) with periodic boundary conditions. The analytical, numerical and experimental results were then compared. The results indicate that the far field methods gave more precise predictions. However, AEH provides more information on the behaviour of the unit-cells. Using this detailed information, it was possible to perform an anisotropy analysis. Furthermore, contrary to the closed-cells, the open-cell numerical methods and analytical models are within the experimental results range.

Keywords: Metal foams; Homogenization; Representative unit-cells; Kelvin Structure; Weaire–Phelan Structure

1. Introduction

Metal foams are a class of materials of increasing interest, that combine a very exciting set of properties that make them interesting for applications in a wide variety of sectors [1]. In order to spread the use of these materials on engineering applications, it is required a detailed understanding of their mechanical properties and behaviour. These properties depend on the characteristics of the base material, such as relative density and morphological parameters (i.e. pore size, type of foam which depend on the manufacturing process used). The relationships between the properties, size of the pores and the foam density have been extensively studied using experimental studies [2,3]. In general, real foams exhibit a high variability in cell sizes and shapes, and wall thickness that influences their elastic properties [4]. An extensive number of uni-axial loading, bending, fatigue experiments were already performed to understand the plastic deformation [5–9]. To properly characterize these heterogeneous materials, it is necessary to complement the experimental studies with numerical and analytical studies. The scope of this work is to define numerical procedures to predict the elastic behaviour of metal foams. First by modelling the representative geometries of the foams, secondly by using numerical analysis software to implement the different homogenization methods and lastly by comparing the predicted values with the analytical models obtained from prior works and experimental results.

2. Numerical method

2.1. Far field method with single freedom constraints

Here the unit-cell behaves as if the strain or stress field arise from the macrostructural problem, often called far field stress or strain. The effective elastic properties of the material are determined by using Hook's law. This method was divided into two approaches based on the type of load case used on the unit-cell. That was, either **(i)** an imposed force F_y over the surface area A or **(ii)** a prescribed displacement ΔL_y over the length L, the effective Young's modulus E is given by

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$$E = \frac{F_y L}{\Delta L_y A} \tag{1}$$

The degrees of freedom for three faces of the unitcell are constrained and the displacement in these faces is only tangential, therefore enforcing the condition that a pair of faces must stay parallel throughout the deformation history [10].

2.2. Asymptotic expansion homogenization

To ensure cell-to-cell continuity, the opposite geometrical boundaries of a given cell have to be identical, both for the original and deformed states. The periodicity of the deformed unit-cell in this method is granted by the applying periodic boundary conditions [11].

The microscale problem is solved on two main steps. The first is associated to the calculation of the characteristic displacement field tensor $\boldsymbol{\chi}$. The element strain and stress matrices are given by $\varepsilon = \mathbf{Bu}$ and $\sigma = \mathbf{DBu}$, respectively where $\mathbf{B}$ is the element strain matrix, $\mathbf{u}$ is the vector of nodal displacements, and $\mathbf{D}$ is the matrix of material properties. Therefore the calculation of the corrector matrix $\boldsymbol{\chi}$ [12,13]

$$\int_{Y^e} \mathbf{B^T D B} \mathrm{d}Y \boldsymbol{\chi} = \int_{Y^e} \mathbf{B^T D} \mathrm{d}Y = \mathbf{F^D} \tag{2}$$

where the script e corresponds to element quantities associated with the discretized finite element domain of the unit-cell, namely the body Y^e. The corrector is a matrix on contrary to the case of displacements in conventional elasticity. The second term of Eq. (2) is made of the columns of the load matrix $\mathbf{F^D}$ [12,14].

In the second step of the microscale problem solving, the matrix $\boldsymbol{\chi}$ is used to correct the homogenized elasticity properties, accounting for the microscale material distribution effect on the volume average. For the AEH approach, the homogenized elasticity matrix $\mathbf{D^h}$ correspond to,

$$\mathbf{D^h} = \sum_{k=1}^{n_e} \frac{Y^k}{Y} \mathbf{D}^k (\mathbf{I} - \mathbf{B}^k \boldsymbol{\chi}^k) \tag{3}$$

where Y^k is the volume of element k, Y the total geometry volume and $\mathbf{I}$ the identity matrix. If $\boldsymbol{\chi} = 0$, this equation becomes the classical volume average of the elastic properties of the microscale elements.

The inverse of the constitutive matrix $\mathbf{D^h}$ results in the flexibility matrix $\mathbf{S^h}$,

$$\mathbf{S^h} = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & \frac{1}{E_{11}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{12}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix}$$

The $\mathbf{S}_{11}$, $\mathbf{S}_{22}$ and $\mathbf{S}_{33}$ components of $\mathbf{S^h}$ correspond to the inverse elastic modulli of the material in each orthogonal direction [12].

2.3. Modelling cellular materials

When the foam relaxes, tends to a local minimum surface free-energy density, minimizing the surface tension and increasing the surface area per unit volume [15]. The understanding of this process helps the modelling of the representative geometries. To describe the foams geometry one can use: Space-filling polyhedra, Tessellation-based models or image based models which are more accurate representations of the foams. This work uses Space-filling polyhedra as geometrical representation of the foams to characterize the elastic behaviour. The reliability of the unit-cells is usually evaluated by two assumptions, the Plateau's law of equilibrium and the Matzke studies [16–19]. Based on this, the geometries used are, the Kelvin and Weaire–Phelan unit-cells. Fig. 1 shows the modelled geometries used. Even though open-cells foams are not governed by the Plateau's law, it was used the same geometry definition for open and closed-cell foams.

The unit-cells were modelled in CATIA V5R15, with the aid of Ken Brakke's Surface Evolver free ware [20]. The far

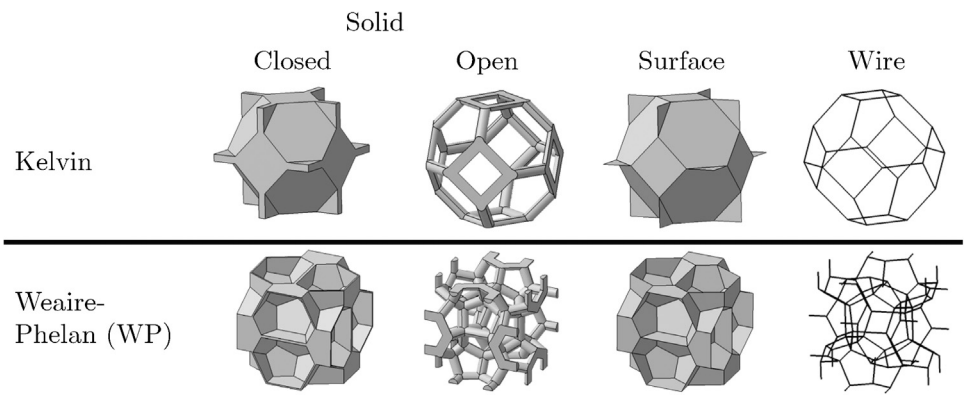

Fig. 1. The figure compiles all RUCs type used in this work.