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Research Paper

A coupled thermo-hydro-mechanical finite element formulation of one-dimensional beam elements for three-dimensional analysis

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ABSTRACT

Finite element (FE) analysis in geotechnical engineering often involves entities which can be represented as one-dimensional elements in three-dimensions (e.g. structural components, drains, heat exchanger pipes). Although structural components require an FE formulation accounting only for their mechanical behaviour, for the latter two examples, a coupled approach is necessary. This paper presents the first complete coupled thermo-hydro-mechanical FE formulation for one-dimensional beam elements for three-dimensional analysis. The possibility of deactivating each of the systems enables simulation of both coupled and uncoupled behaviour, and hence a range of engineering problems. The performance of these elements is demonstrated through various numerical simulations.

1. Introduction

Many geotechnical problems involve structural components such as retaining walls, tunnel linings, anchors, props, etc. which must be included in the finite element (FE) analysis. Although these components can be represented by continuum finite elements, this approach may result in a large number of elements increasing the computational effort, or in elements with unacceptable aspect ratios due to their geometries. Instead, they can be modelled using specific finite elements which are formulated by reducing one or more dimensions of the structural component to zero, and hence represent it as a line or a surface with zero thickness [1].

This study focuses on structural components that can be modelled as line (i.e. one-dimensional) elements in three-dimensional (3D) analysis. Examples of such components include anchors, props, beams, columns, piles, sand columns as well as drains. Moreover, the recent increase in the use of systems that utilise geothermal energy (i.e. ground source energy systems) has created the need for accurate prediction of the temperature field in the ground. This requires the inclusion in the numerical model of the heat exchanger pipes through which a fluid is circulated facilitating the transfer of thermal energy between the building and the ground. Due to their small diameters, they can also be represented as one-dimensional elements in 3D. Clearly, in the case of structural elements such as anchors, props, beams or columns, their mechanical behaviour is of interest to geotechnical engineers, and therefore, the FE analysis must solve the force equilibrium equations. However, in order to model drains or sand columns, a coupled hydro-mechanical (HM) FE formulation is required, whereas a coupled thermo-hydraulic (TH) formulation is necessary for heat exchanger pipes. Furthermore, thermal drains, which have been shown to enhance the consolidation rates in soft soils by imposing high temperatures (e.g. [2,3]), require the solution of coupled thermo-hydro-mechanical equations (THM). It should be highlighted that in all the above examples, the one-dimensional elements are used within a soil mass simulated using 3D solid elements.

The FE software PLAXIS 3D [4] uses one-dimensional elements, referred to as beam elements, for modelling structural components in 3D. These are 3-noded elements with five degrees of freedom per node, i.e. three displacements (one axial and two transverse) and two rotations. As the rotation degree of freedom around the element's axis is not considered, these beams cannot sustain torsional moments. One-dimensional beam elements with an additional rotational degree of freedom, which allows for torsion, are available in the FE programs ABAQUS [5] and CRISP [6]. Moreover, one-dimensional elements (referred to as truss elements in ABAQUS and bar elements in CRISP), which have only displacement degrees of freedom, and hence, transmit only axial forces, have also been developed. However, it should be noted that these FE formulations are purely mechanical, and therefore, the one-dimensional elements cannot be used in problems where their thermal or hydraulic behaviour is important.

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In order to model drains, Hird et al. [7] and Russell [8] developed a 3-noded line drainage element for use in plane strain and axisymmetric consolidation analysis in CRISP. However, as this element has a zero cross-sectional area, it is unclear whether its use can be extended to the modelling of the mechanical behaviour of structural components. Furthermore, the element cannot simulate the fully coupled HM behaviour as the off-diagonal coupling terms in this HM formulation are zero. Finally, the formulation has not been extended for 3D analysis. In PLAXIS, drains are simulated using a seepage boundary which permits water to leave the mesh at atmospheric pressure [4].

The conductive-advective flows in heat exchanger pipes (i.e. coupled TH problems) have been simulated in 3D with COMSOL Multiphysics (e.g. [9–11]) using a pipe interface which solves the continuity, fluid momentum and energy balance equations [12]. However, to the authors' knowledge, the coupled THM FE formulation of one-dimensional elements in 3D analysis has not been developed.

This paper presents a complete fully coupled THM FE formulation of one-dimensional beam elements for 3D analysis which has been implemented into the bespoke FE software – the Imperial College Finite Element Program (ICFEP, Potts and Zdravković [1]). The elements may be either 2- or 3-noded with each node possessing several degrees of freedom: three displacements, three rotations, as well as pore fluid pressure and temperature. The mechanical formulation of these new elements is based on that for Mindlin's beam elements for two-dimensional (2D) analysis developed by Day and Potts [13] which has been shown to be valid for both straight and the more problematic curved beams. As the extension of this formulation to 3D involves additional displacement and rotation degrees of freedom, the new beam elements can transmit axial and shear forces, as well as bending and torsional moments. The coupled THM formulation is based on that for continuum elements presented by Cui et al. [14], and therefore, is compatible with other finite element types in ICFEP, allowing modelling of the interactions between the one-dimensional elements and other structural components and/or the surrounding medium. Additionally, in order to overcome problems associated with modelling of highly advective flows, a Petrov-Galerkin finite element method (FEM) instead of the Galerkin FEM may be used when advection dominates heat transfer along the element. It should be noted that any of the three systems of the coupled THM formulation (i.e. mechanical, hydraulic and thermal) may be disabled such that the beam elements can be used in a variety of problems involving coupled HM, TH or TM, or uncoupled mechanical, hydraulic or thermal behaviour. To demonstrate the performance of the new one-dimensional beam elements, a series of validation exercises is presented in this paper. Finally, it should be noted that one-dimensional bar elements have also been developed, although, as they differ from beam elements only in terms of their mechanical behaviour, their formulation is presented briefly in Appendix A. The sign convention adopted throughout this paper is such that tensile stresses, strains and forces are positive.

2. Coupled THM formulation for a one-dimensional beam element

2.1. Mechanical behaviour

Mechanically, the one-dimensional beam elements have six degrees of freedom per node (three displacements and three rotations), and transmit not only bending moments, axial and shear forces but also a torsional moment. Fig. 1 shows a 3-noded beam element of length l in 3D and defines its six degrees of freedom: displacements u, v and w in the global x, y and z directions, respectively, and rotations θ_x, θ_y and θ_z about the global x, y and z axes, respectively. The sign convention for rotations adopted in this paper follows the right-hand rule as illustrated in Fig. 1.

For convenience, the global nodal displacements and rotations are defined as two separate vectors:

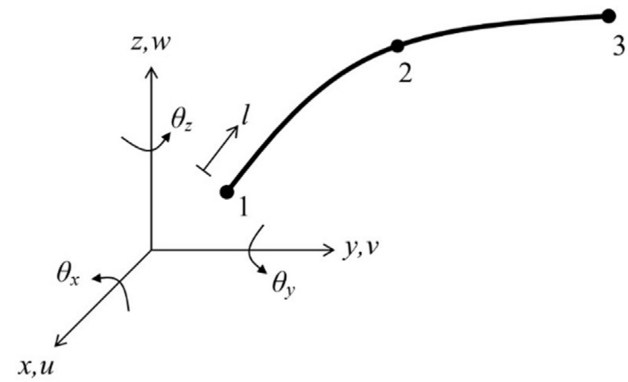

Fig. 1. 3-noded beam element in 3D global coordinate system.

$$\{\delta\} = \{u \quad v \quad w\}^T \tag{1}$$

$$\{\theta\} = \{\theta_x \quad \theta_y \quad \theta_z\}^T \tag{2}$$

Additionally, the following vectors are defined:

$$\{\delta'\} = \left\{\frac{\partial u}{\partial l} \quad \frac{\partial v}{\partial l} \quad \frac{\partial w}{\partial l}\right\}^T \tag{3}$$

$$\{\theta'\} = \left\{\frac{\partial \theta_x}{\partial l} \quad \frac{\partial \theta_y}{\partial l} \quad \frac{\partial \theta_z}{\partial l}\right\}^T \tag{4}$$

2.1.1. Constitutive equations

Under isothermal conditions, the relationship between the total strains and element forces and moments is described by the constitutive equation [1]:

$$\{\Delta\sigma\} = [D]\{\Delta\varepsilon_\sigma\} \tag{5}$$

where for one-dimensional beam elements:

$$\{\Delta\varepsilon_\sigma\} = \{\Delta\varepsilon_{a,\sigma} \quad \Delta\gamma_1 \quad \Delta\gamma_2 \quad \chi_T \quad \Delta\chi_1 \quad \Delta\chi_2\}^T \tag{6}$$

$$\{\Delta\sigma\} = \{\Delta F_a \quad \Delta S_1 \quad \Delta S_2 \quad \Delta M_T \quad \Delta M_1 \quad \Delta M_2\}^T \tag{7}$$

$\{\Delta\varepsilon_\sigma\}$ is the vector of incremental mechanical (or total under isothermal conditions) strains whose components are: $\varepsilon_{a,\sigma}$ – axial strain, γ_1 and γ_2 – shear strains, χ_T – torsional strain, χ_1 and χ_2 – bending strains, whereas $\{\Delta\sigma\}$ is the vector of incremental forces and moments whose components are: F_a – axial force, S_1 and S_2 – shear forces, M_T – torsional moment, M_1 and M_2 – bending moments. These components are related to three local axes, one of which is along the beam and the remaining two are perpendicular to the beam. For a beam with linear elastic behaviour, the constitutive matrix, $[D]$, is defined as:

$$[D] = \begin{bmatrix} EA_c & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{s1}GA_c & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{s2}GA_c & 0 & 0 & 0 \\ 0 & 0 & 0 & GJ_T & 0 & 0 \\ 0 & 0 & 0 & 0 & EI_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & EI_2 \end{bmatrix} \tag{8}$$

where E is the Young's modulus, A_c is the cross-sectional area, k_{s1} and k_{s2} are the shear correction factors, G is the shear modulus, J_T is the torsional constant and I_1 and I_2 are the second moments of area. Note that for beams with cross-sections which are symmetric with respect to the two axis (i.e. circular or square), $k_{s1} = k_{s2} = k_s$ and $I_1 = I_2 = I$ in Eq. (8). The torsional constant, J_T, describes the torsional stiffness of the beam, and its value depends on the shape of the cross-section of the beam.

In a coupled THM problem involving a two-phase porous medium (e.g. soil), the mechanical behaviour of the material is affected by changes in pore fluid pressure, as well as temperature. In order to formulate the equations governing the mechanical behaviour, the