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# On regular and new types of codes for location-domination

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ABSTRACT

Identifying codes and locating-dominating codes have been designed for locating irregularities in sensor networks. In both cases, we can locate only one irregularity and cannot even detect multiple ones. To overcome this issue, self-identifying codes have been introduced which can locate one irregularity and detect multiple ones. In this paper, we define two new classes of locating-dominating codes which have similar properties. These new locating-dominating codes as well as the regular ones are then more closely studied in the rook's graphs and binary Hamming spaces.

In the rook's graphs, we present optimal codes, i.e., codes with the smallest possible cardinalities, for regular location-domination as well as for the two new classes. In the binary Hamming spaces, we present lower bounds and constructions for the new classes of codes; in some cases, the constructions are optimal. Moreover, one of the obtained lower bounds improves the bound of Honkala et al. (2004) on codes for locating multiple irregularities.

Besides studying the new classes of codes, we also present record-breaking constructions for regular locating-dominating codes. In particular, we present a locating-dominating code in the binary Hamming space of length 11 with 320 vertices improving the earlier bound of 352; the best known lower bound for such code is 309 by Honkala et al. (2004).



## 1. Introduction

Sensor networks are systems designed for environmental monitoring. Various location detection systems such as fire alarm and surveillance systems can be viewed as examples of sensor networks. For location detection, a sensor can be placed in any location of the network. The sensor monitors its neighbourhood (including the location of the sensor itself) and reports possible irregularities such as a fire or an intruder in the neighbouring locations. Based on the reports of the sensors, a central controller attempts to determine the location of a possible irregularity in the network. Usually, the aim is to minimize the number of sensors in the network. More explanation regarding location detection in sensor networks can be found in [4,12,16].

A sensor network can be modelled as a simple and undirected graph $G = (V(G), E(G)) = (V, E)$ as follows: the set of vertices $V$ of the graph represents the locations of the network and the edge set $E$ of the graph represents the connections between the locations. In other words, a sensor can be placed in each vertex of the graph and the sensor placed in the vertex $u$ monitors $u$ itself and the vertices neighbouring $u$. Besides being simple and undirected, we assume that the graphs in this paper are connected and have order of at least two. In what follows, we present some basic terminology and notation regarding graphs. The *open neighbourhood* of $u \in V$ consists of the vertices adjacent to $u$ and it is denoted by $N(u)$. The *closed neighbourhood* of $u$ is defined as $N[u] = \{u\} \cup N(u)$. A nonempty subset $C$ of $V$ is called a *code* and the elements of the code

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are called *codewords*. In this paper, the code $C$ represents the set of locations where the sensors have been placed on. For the set of sensors monitoring a vertex $u \in V$, we use the following notation:

$$I(u) = N[u] \cap C.$$

In order to emphasize the graph $G$ and/or the code $C$, we sometimes write $I(u) = I(C; u) = I(G, C; u)$. We call $I(u)$ the *identifying set* (or the *I-set*) of $u$. The notation of identifying set can also be generalized for a subset $U$ of V as follows:

$$I(U) = \bigcup_{u \in U} I(C; u).$$

Here we also use the notation $I(U) = I(C; U) = I(G, C; U)$.

As stated above, a sensor $u \in V$ reports that an irregularity has been detected if there is (at least) one in the closed neighbourhood $N[u]$. In what follows, we divide into two different situations depending on the capability of a sensor to distinguish whether the irregularity has been spotted in the location of the sensor itself or in its (open) neighbourhood. More precisely, we have the following two cases:

(i) In the first case, we assume that a sensor $u \in V$ reports 1 if there is an irregularity in $N[u]$, and otherwise it reports 0.
(ii) In the second case, we assume that a sensor $u \in V$ reports 2 if there is an irregularity in $u$, it reports 1 if there is one in $N(u)$ (and none in $u$ itself), and otherwise it reports 0.

Assume first that the sensors work as in (i). Notice then that if the sensors in the code $C$ are located in such places that $I(C; u)$ is nonempty and unique for all $u \in V$, then an irregularity in the network can be located by comparing $I(C; u)$ to identifying sets of other vertices. This leads to the following definition of *identifying codes*, which were first introduced by Karpovsky et al. in [11]. For various papers regarding identification and related problems, we refer to the online bibliography [13].

**Definition 1.** A code $C \subseteq V$ is *identifying* in $G$ if for all distinct $u, v \in V$ we have $I(C; u) \neq \emptyset$ and

$$I(C; u) \neq I(C; v).$$

An identifying code $C$ in a finite graph $G$ with the smallest cardinality is called *optimal* and the number of codewords in an optimal identifying code is denoted by $\gamma^{ID}(G)$.

Let $C$ be an identifying code in $G$. By the definition, the identifying code $C$ works correctly if there is simultaneously at most one irregularity in the network. However, using the identifying code $C$, we cannot locate or even detect more than one irregularity in the network. Indeed, for example, consider the graph $G$ in Fig. 1 and the code $C = \{a, b, c\}$ in the graph. Clearly, $C$ is an identifying code in $G$. However, all the sensors $a$, $b$ and $c$ are alarming if there is a single irregularity in $b$, or multiple ones in $d$, $e$ and $f$. Hence, no distinction can be made between these two cases. Thus, we might determine a false location and more disturbingly not even notice that something is wrong. To overcome this problem, in [7], self-identifying codes, which are able to locate one irregularity and detect multiple ones, were introduced. (Notice that in the original paper self-identifying codes are called $1^+$-identifying.) The formal definition of self-identifying codes is given as follows.

**Definition 2.** A code $C \subseteq V$ is called *self-identifying* in $G$ if the code $C$ is identifying in $G$ and for all $u \in V$ and $U \subseteq V$ such that $|U| \geq 2$ we have

$$I(C; u) \neq I(C; U).$$

A self-identifying code $C$ in a finite graph $G$ with the smallest cardinality is called *optimal* and the number of codewords in an optimal self-identifying code is denoted by $\gamma^{SID}(G)$.

In addition to [7], self-identifying codes have also been previously discussed in [9,10]. Separately in these papers, two useful characterizations have been presented for self-identifying codes.

**Theorem 3** (*[7,9,10]*)**.** *Let $C$ be a code in $G$. Then the following statements are equivalent:*

(i) *The code $C$ is self-identifying in $G$.*
(ii) *For all distinct $u, v \in V$, we have $I(C; u) \setminus I(C; v) \neq \emptyset$.*
(iii) *For all $u \in V$, we have $I(C; u) \neq \emptyset$ and*

$$\bigcap_{c \in I(C;u)} N[c] = \{u\}.$$

As stated earlier, self-identifying codes can locate one irregularity and detect multiple ones. Besides that, the characterization (iii) of the previous theorem also gives another useful property for self-identifying codes. Namely, the location of an irregularity can be determined without comparison to other identifying sets, since for all $u \in V$ the neighbourhoods of the codewords in $I(u)$ intersect uniquely in $u$.

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