Discrete Applied Mathematics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Discrete Applied Mathematics

journal homepage: www.elsevier.com/locate/dam

Conditional diagnosability of multiprocessor systems based on complete-transposition graphs[☆]

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ARTICLE INFO

Article history:
Received 28 October 2016
Received in revised form 22 May 2017
Accepted 28 March 2018
Available online xxxx

Keywords:
Cayley graphs
Fault diagnosis
Conditional diagnosability
Complete-transposition graphs

ABSTRACT

Diagnosability is an important parameter to measure the ability of diagnosing faulty processors of a multiprocessor system. Conditional diagnosability is a realistic improvement of classical diagnosability under the condition that every processor has at least one fault-free neighboring processor. Complete-transposition graphs are proposed to be potential competitive network models of hypercubes as well as star graphs. In this paper, we show that the conditional diagnosability of the complete-transposition graph CT_n under the MM* model is $\frac{3}{2}n(n-1)-6$ for $n \geq 7$, while the conditional diagnosability of CT_n under the PMC model is $2n(n-1)-9$ for $n \geq 5$.

1. Introduction

Due to continuous advances in technology, a multiprocessor system may consist of hundreds or thousands of processors. Some of these processors may be defective while the system is in operation. In order to maintain reliability, the system should be able to identify faulty vertices and replace them with fault-free ones. The process of identifying faulty processors in a multiprocessor system by analyzing the outcomes of available inter-processor tests is called *system-level diagnosis*, and the *diagnosability* of the system refers to the maximum number of faulty vertices that can be identified by the system.

For the purpose of self-diagnosis of a system, several diagnosis models have been proposed for a long time. Two of the most important models are the MM model [18] and the PMC model [19]. The MM model is also called comparison diagnosis model, in which, the diagnosis is performed by sending the same input from a processor (node) to each pair of its distinct neighbors, and then comparing their outcomes. Sengupta and Dahbura [20] suggested the MM* model which is a modification of the MM model. In the MM* model, every processor must test any two processors if it is adjacent to them. Only a fault-free processor can guarantee reliable outcome, and the output are identical if the two processors, which are adjacent to it, are fault-free; and distinct otherwise. The MM* model was adopted to evaluate fault tolerance of interconnection networks [5,22]. In the PMC model, each node u is able to test each of its adjacent nodes, say v, where u is called the tester and v is called the tested node. The outcome of a test performed by a fault-free tester is 1 (resp., 0) if the tested node is faulty (resp., fault-free); however, the outcome of a test performed by a faulty tester is unreliable. The PMC model was adopted in [2–4,14,27].

☆ This work is supported by National Natural Science Foundation of China (Nos. 11301217, 61572010, 61602118, 61702100, 11571139), New Century Excellent Talents in Fujian Province University (No. JA14168, JAT170118) and Natural Science Foundation of Fujian Province, China (Nos. 2018J01419, 2015J01017, 2017J01738).

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https://doi.org/10.1016/j.dam.2018.03.079
0166-218X/

Given a multiprocessor system, it is not possible to determine whether some processor u is fault-free or not, if all the neighbors of processor u are faulty. In this case, Lai et al. [14] proposed the conditional faulty set, which is a special faulty set that does not contain all of neighbors of any vertex in a network. The conditional diagnosability is a metric that can give the maximum number of conditional faulty set that the system is guaranteed to identify. Many researchers have studied the conditional diagnosability of different networks under different models. The conditional diagnosability of a k-ary n-cubes Q_k^n $(n \geq 4, k \geq 4)$ is $6n-5$ under the MM* model [11] and $8n-7$ under the PMC model [4]. The conditional diagnosability of the n-dimensional hypercube Q_n is $3n-5$ for $n \geq 5$ under the MM* model [13] and $4n-7$ under the PMC model [14]. The conditional diagnosability of folded hypercube FQ_n [12] is $3n-2$ for $n \geq 5$ under the MM* model. The conditional diagnosability of star graph S_n [26] is $3n-8$ for $n \geq 4$ under the MM* model and $8n-21$ for $n \geq 5$ under the PMC model [3]. The conditional diagnosability of bubble-sort graph B_n [27] is $4n-11$ for $n \geq 4$ under the PMC model. The conditional diagnosability of bubble-sort star graph BS_n is $6n-15$ for $n \geq 6$ under the MM* model and $8n-21$ for $n \geq 5$ under the PMC model [7]. Other types of diagnosability, such as pessimistic diagnosability [6] and strong diagnosability [8–10], are also discussed.

Traditionally, the interconnection network topology has been centered on hypercube, since it has very simple and abundant topological properties. The complete-transposition graphs were introduced by Leighton [16], and are considered as very promising network topologies for interconnection network due to its salient features such as symmetry, recursive scalability, and efficient routing, etc. [1,15]. Comparing to hypercubes and star graphs, the class of complete-transposition graphs were believed to have better performance in terms of degree, diameter, connectivity, and fault tolerance, etc. Lakshmivarahan et al. [15] proved that the n-dimensional complete-transposition graph, denoted by CT_n, has both vertex and edge transitivity. It is well known that every edge transitive graph is maximally connected [25]. Hence $\kappa(CT_n) = \frac{1}{2}n(n-1)$. Recently, Wang et al. [23,24] have established the 2-restricted vertex (edge)-connectivity, 3-restricted vertex (edge)-connectivity, R^1-vertex-connectivity and R^2-vertex-connectivity of CT_n.

In this paper, we will explore the conditional diagnosability of the complete-transposition graph under the MM* Model and the PMC Model based on fault tolerance analysis of corresponding topological structure. In detail, we show that the conditional diagnosability of the complete-transposition graph CT_n under the MM* model is $\frac{3}{2}n(n-1)-6$ for $n \geq 7$, while the conditional diagnosability of CT_n under the PMC model is $2n(n-1)-9$ for $n \geq 5$. The rest of this paper is organized as follows. Section 2 introduces some necessary definitions and notations. Section 3 explores the conditional diagnosability of the complete-transposition graphs under the MM* Model. Section 4 demonstrates the conditional diagnosability of complete-transposition graphs under the PMC Model. Section 5 concludes the paper.

2. Preliminaries

A multiprocessor system can be represented by a graph $G = (V(G), E(G))$, where the set of vertices $V(G)$ represents processors and the set of edges $E(G)$ represents communication links between processors. Throughout this paper, we focus on undirected graphs without loops and follow [21] for graph theoretical definitions and notations.

For $v \in V(G)$, the degree of v, written by $d(v)$, is the number of edges incident with v. Let $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree of G, respectively. If $\Delta(G) = \delta(G)$, then the graph is regular. The set of neighbors of a vertex v in G is denoted by $N_G(v)$. For a subset S of V, $G[S]$ is a subgraph of G induced by S. The neighborhood set of S in G is defined as $N_G(S) = (\cup_{u \in S} N_G(u)) \setminus S$. We will use $G - S$ to denote the subgraph $G[V - S]$. The minimum size of a vertex set $S \subset V(G)$ such that the graph $G - S$ is disconnected or has only one vertex is the connectivity of G, denoted by $\kappa(G)$. The symmetric difference of $F_1 \subset V(G)$ and $F_2 \subset V(G)$ is defined as the set $F_1 \triangle F_2 = (F_1 - F_2) \cup (F_2 - F_1)$. Let $cn(G)$ be the maximum number of common neighbors between any two vertices in G.

In the MM model, a self-diagnosable system of a graph G is often represented by a multigraph $M(V, L)$, where V and L are the vertex set of G and the labeled edge set, respectively. $(u, v; w)$ is defined as a labeled edge, if both of vertices u and v are adjacent to w, which implies that u and v are being compared by w. Since a pair of vertices may be compared by different vertices, M is a multigraph. For $(u, v; w) \in L$, we use $\delta((u, v; w))$ to denote the result of comparing vertices u and v by w. For w being fault-free, if both of u and v are fault-free, then $\delta((u, v; w)) = 0$; otherwise $\delta((u, v; w)) = 1$. If w is faulty, $\delta((u, v; w))$ may be either 1 or 0, which implies the result is unreliable. The collection of all comparison results in $M(V, L)$, defined as a function $\delta : L \rightarrow \{0, 1\}$, is the syndrome of the diagnosis.

Under the PMC model, a self-diagnosable system of a graph G is often represented by a digraph $D(V, L)$, where V and L are the vertex set of G and the order edge set, respectively. (u, v) is defined as an order edge, if the vertex u is adjacent to v, which implies that u can test v. For $(u, v) \in L$, we use $\delta((u, v))$ to denote the result of testing some vertex v by u. For u being fault-free, if v is fault-free, then $\delta((u, v)) = 0$; otherwise $\delta((u, v)) = 1$. If u is faulty, $\delta((u, v))$ may be either 1 or 0, which implies the result is unreliable. The collection of all comparison results in $D(V, L)$, defined as a function $\delta : L \rightarrow \{0, 1\}$, is the syndrome of the diagnosis. This diagnosis model assumes that each node u tests the other whenever they are adjacent to it. A subset $F \subseteq V(G)$ is *compatible* with a syndrome δ if the syndrome can arise the circumstance that all vertices in F are faulty while all vertices in $V(G) - F$ are fault-free. Since a faulty comparator w may return an unreliable result, a faulty set F may produce different syndromes. Let $\delta(F)$ denote the set of all syndromes that is compatible with F. A system is said to be diagnosable if for every syndrome δ, there is a unique $F \subseteq V(G)$ that is compatible with δ. It is called t-diagnosable if the system is diagnosable as long as the size of faulty set does not exceed t. The maximum number of t such that the graph G is t-diagnosable is called the *diagnosability* of G, written as $t(G)$.