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Improved modeling of free power-law liquid sheets by weighted-residual approximations

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ABSTRACT

The linear temporal instability of planar power-law liquid sheets surrounded by viscous gas medium is investigated theoretically for symmetric disturbances. The linear stability analysis is improved by method of the weighted residual approach using an appropriate projection basis, instead of the traditional Kármán's momentum integral method which is known to make the resulting model inaccurate near the stability threshold. Two different ways are used to deal with linear stability analysis with the existence of the viscous gas medium: inviscid gas pressure approximation and spectral collocation method. A system of two equations in relation to the film thickness h and the local flow rate q is obtained through the first way and then the first-order dispersion equation is deduced. The results are analyzed to investigate how the power-law index, consistency coefficient, gas boundary-layer thickness and other rheological parameters affect the atomization mechanism of planar power-law sheets. Meanwhile, comparisons of the results obtained by the two ways have been made to verify rationality of the inviscid gas pressure approximation. Furthermore, the competition between the power-law characteristic property and aerodynamic instability on sheet instability has been examined. Finally, the theoretical calculations have been compared with previous experimental results and show relatively good agreement.

1. Introduction

In rocket engine applications, gel propellants (particles suspended in gel flammable liquids) have better performance and higher safety than conventional liquid and solid propellants. For the gel propellant, when exerted a stress, the response of it is usually nonlinear and presents a shear-thinning behavior. When the gel propellant is subjected to steady-state shear rheological testing, the relationship between its shear viscosity μ and shear rate $\dot{\gamma}$ can be fitted by a power function $\mu = k\dot{\gamma}^{n-1}$. Hence, it is also called the power-law fluid in practical engineering.

Up to now, a lot of researchers have devoted themselves to investigating flow instability of power-law fluids. However, most of their studies are based on experimental methods. Chojnacki and Feikema (1994) used Carbopol 941 (a kind of power-law fluid) to investigate the influence of jet velocity and the impingement angle on breakup of turbulent jets. They found that the relatively long ligaments exhibited by the gel spray suggested a polymeric effect that was attributed to the stronger inter-molecular forces. However, their experimental results about jet instability were restricted to limitation of low jet velocity, without taking into account the generation of droplets. Lee and Koo (2010) focused on the breakup process, wave development of ligament and liquid sheets formed by impinging jets with various gelling agent contents. The experimental results showed the significance of rheological properties of power-law fluids, but were mainly analyzed qualitatively. Jayaprakash and Chakravarthy (2003) conducted an experimental study of a spray created by the impingement of two jets of metallized gelled fuel. They pointed out that the mean droplet size is lower at a higher included angle and injection pressure due to the high shear rate in the injector orifice and impingement point. Baek et al. (2011) used doublet like-on-like impinging jets to investigate the spray and atomization behavior of C934 Carbopol gels with or without SUS304 nanoparticles and a comparison with the spray behavior of water was also made. Besides, they also measured the drop SMD using an image processing method and concluded that the elasticity of gels appears to reduce the asymptotic drop size. Yang et al. (2012) performed experiments with injectors of different configurations and used a high-speed camera to show detailed information of the liquid sheet breakup process. In their study, three impinging injectors were tested under different Weber numbers to measure their breakup lengths and critical wavelengths, aimed at validating the sheet breakup model.

In order to gain the insight into the physics of the atomization and spray of gel materials, some researchers tried to conduct

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theoretical stability analysis of the gel with the power-law model. Joo (1994) first included viscoelastic effects along with power-law fluid effects in the case of thin films flowing down walls. They used the Oldroyd four-constant model to incorporate elastic and shear-thinning effects. Lin and Hwang (2000) used long-wave theory to investigate nonlinear stability of power-law liquid films down an inclined plane. They found that shear-thinning liquid films are more unstable than Newtonian's, and only at larger power-law index do the supercritical region and unconditional stable region exist when disturbed modes are near the neutral stability curve. Balmforth et al. (2003) adopted a lubrication-style approximation of the governing fluid equations to investigate interfacial instability of superposed layers of fluid flowing down an inclined plane. Their results indicated that instabilities saturate at low amplitude beyond onset and form steady wavetrains. Dandapat and Mukhopadhyay (2001) researched waves that occur at the surface of a power-law fluid film flowing down an inclined plane. Later, they investigated waves that occur at the surface of a falling film of thin power-law fluid on a vertical plane (Dandapat and Mukhopadhyay, 2003). By method of integral relations, they derived an evolution equation representing two waves equations under long wave approximations. Miladinova et al. (2004) considered the problem of a thin layer of a power-law liquid falling down an inclined plate and solved numerically the corresponding nonlinear evolution equation for the film thickness in a periodic domain. Their results showed that the free-surface evolution was similar to that for a Newtonian liquid, but the shape and amplitude of the permanent wave were influenced strongly by the non-Newtonian fluid behavior. Chen (2007) examined the effect of Marangoni convection on the flow and heat transfer within a power-law liquid film on an unsteady stretching sheet. He found that the velocity and temperature distributions in the film are affected significantly by the thermally induced Marangoni convection adjacent to the free surface. Cheng and Liu (2009) employed the long-wave perturbation method to investigate the hydromagnetic stability of a thin electrically-conductive power-law liquid film flowing down the external surface of a vertical cylinder in a magnetic field. The stability criteria have been discussed both theoretically and numerically and corresponding stability diagrams have been derived. Amaouche et al. (2012) dealt with modeling of a power-law fluid film flowing down an inclined plane for small to moderate Reynolds numbers. They pointed out that long waves of Benney type (Benney, 1966) allow description of the flow development near criticality; however, when the convective effects become significant, the Benney equation would lose its physical relevance, due to the appearance of shorter waves. Therefore, they proposed to improve the modeling through a combination of the lubrication theory and the weighted residual approach. Chang et al. (2013) investigated temporal instability of a power-law liquid jet injected into a static inviscid gas medium for axisymmetric disturbances. The results show different instability characteristics for different modes (Rayleigh mode and Taylor mode) and power law exponents.

For non-Newtonian liquid sheets, there usually exist two modes in linear analysis, i.e., sinuous mode (antisymmetric mode) and varicose mode (symmetric mode). Liu et al. (1998) investigated both symmetric and antisymmetric instabilities for viscoelastic planar liquid sheets and discovered that antisymmetric disturbances always prevail over symmetric disturbances for viscoelastic liquid sheets. Alleborn et al. (1999) conducted a linear stability analysis of a viscoelastic annular liquid sheet for both even and odd modes. In their study, the most dangerous mode was localized and characterized. Davalos-Orozco (1999) examined linear thermocapillar instability of a moving viscous liquid sheet through an inviscid gas and found that the sinuous mode has more possibilities to appear than the varicose mode under large enough Marangoni numbers and Ohnesorge numbers. Notice that when including different effects to the liquid sheets, it is possible to get, for some magnitudes of the parameters, the symmetric mode as the prevailing one, and for other magnitudes, the antisymmetric mode of instability is the most unstable one. Recently, Tong et al. (2014) explored thermocapillar instability of a two-dimensional viscoelastic planar liquid sheet surrounded by a gas medium. Only the sinuous mode was considered for the sheet instability. Wang et al. (2015) made a second-order weakly nonlinear analysis of the temporal instability of two-dimensional planar viscoelastic liquid sheets moving in an inviscid gas, only for the linear sinuous mode. On basis of this study, Xie et al. (2016) considered the influence of the unrelaxed stress tension on the nonlinear instability of viscoelastic sheets using a weakly nonlinear method. For the first-order solutions, both the sinuous and varicose dispersion relations were obtained. However for the nonlinear solutions, only the second-order analysis for the sinuous mode was conducted. They concluded that the second-order disturbance is varicose, which explains the final breakup of the viscoelastic liquid sheet.

Throughout the studies above, most of them are based on flow stability for viscoelastic fluids or for power-law fluid films along the inclined plane and reports on atomization mechanism of free power-law sheets are rare. Yang et al. (2015) carried out a linear stability analysis for a power-law planar liquid sheet by method of an integral boundary layer (IBL) approach, which was first introduced by Shkadov (1967). However, this method IBL was commented by Ruyer-Quil and Manneville (2000) that the IBL approach has the too rough character of the consistency condition expressed via the averaging, because the IBL only considers zeroth order in the long wave parameter near criticality. Therefore it is required to remove this drawback with the first order correction of the convective terms near criticality. To improve the modeling process, they proposed weighted residual techniques with polynomials as test functions and finally obtained satisfactory results. Again, Ruyer-Quil et al. (2012) applied weighted residual techniques and average the boundary-layer equations across the film to study the wavy regime of a power-law film flow. It was demonstrated that the in-depth integration of the momentum equation with a uniform weight is now practically a thing of the past, a more careful choice of the weight yields far better results.

In the present study, we first try to use this method to deal with long-wave instability of free power-law liquid sheets. However, for a free liquid sheet, it needs to satisfy more boundary conditions than those for a liquid sheet along an incline. It is difficult to deal with problems for free liquid sheets (especially for power-law fluid) by using WRIBL method. The breakthrough point is to find the similarities between them. Fortunately, we found that for the varicose mode (symmetric disturbances), there is $v = 0$ at $y = 0$ for the free liquid sheets, which is similar to the wall boundary condition $v = 0$ at $y = 0$ for the liquid sheets on an incline. In this case, we can expand the WRIBL method to solve flow instability problems for free power-law liquid sheets under symmetric disturbances. In previous research and other conditions, it has been found that the symmetric or antisymmetric mode of instability may appear in the same problem as the most unstable one. While for the antisymmetric modes, we would strive to break through this difficulty in the future research work.

Therefore, in this paper, a method of a combination of lubrication theory and weighted residual approach is adopted to conduct linear stability analysis and improve modeling of planar power-law sheets with symmetric surface disturbances surrounded by incompressible and viscous gas medium. A two-equation system has been established and effects of the power-law properties have been investigated. And the interaction between the power-law characteristic property and aerodynamic instability on liquid sheets has also been examined. Here in the present study, since the gas out-