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Analysis of thermally developing forced convection heat transfer in a porous medium under local thermal non-equilibrium condition: A circular tube with asymmetric entrance temperature

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ABSTRACT

Thermally developing forced convective heat transfer in a circular tube filled with a fluid-saturated porous medium is analytically investigated under local thermal non-equilibrium (LTNE) condition. In this study, we assume the forced convection within the porous medium obeys Darcy's law, the tube has a constant temperature wall and possess an entrance with variable circumferential temperature distribution. Under these conditions, closed-form exact solutions of the fluid and solid phases temperatures are derived by using the method of separation of variables. The finite element (FE) analysis software COMSOL Multiphysics is employed to numerically model the studied problem. The good agreement between the presented analytical solutions and the FE solutions verifies the validity of the analytical solutions presented. Meanwhile, the consistency of the axisymmetric analytical solutions in the case of uniform entrance temperature with the existing analytical solutions in the literature confirms again the correctness and the reliability of the presented analytical solutions. Given the entrance temperature can be an arbitrary well-defined function, the presented analytical solutions are of general applicability. The subsequent parametric studies are carried out based on the presented analytical solutions. It is demonstrated that the temperature fields are non-axisymmetrical with respect to the circumferential coordinate, which is induced by the asymmetric entrance temperature. Besides, the Péclet number and the Biot number play significant roles in heat transfer. The increase of the Péclet number will result in the transition of thermally developing forced convection to the fully developed regime, while the temperature difference between the fluid and solid phases decreases with increasing the Biot number. Moreover, as a limiting case, the LTNE model degenerates to the LTE model when the Biot number approaches infinity. Our findings are of benefit to provide deep insights into the thermal behavior of forced convection in a porous circular tube with asymmetric entrance temperature under LTNE condition.

1. Introduction

Forced convection heat transfer in a channel or tube filled with a porous medium has attracted much attention stemming from numerous practical applications such as thermal energy storage, electronic cooling, nuclear waste repository, enhanced geothermal system, catalytic reactors, and heat pipes [1–3].

Considering the geometrical symmetry of a channel or tube, the studied problem can be regarded as a two-dimensional symmetric problem provided that the boundary conditions on the walls and entrances are also symmetrical. As to the above two-dimensional symmetric problems, there have been some analytical and numerical solutions. For example, Nield et al. [4] analytically explored the temperature distributions of the thermally developing forced convection in a parallel-plate channel filled by a saturated porous medium with walls at constant temperature under LTNE condition. Later, they extended their work to incorporate the longitudinal conduction and viscous dissipation effects [5].

Yang and Vafai [6] analytically investigated the phenomenon of temperature gradient bifurcation in a porous medium by studying the convective heat transfer process within a channel filled with a porous medium. According to their findings, the phenomenon of temperature gradient bifurcation for the fluid and solid phases with the constant temperature boundary condition could occur over a given longitudinal region. Dukhan and Suleiman [7] numerically studied the thermally developing region in a metal-foam channel with open pores and high porosity.

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Nomenclature

a	dimensionless coefficient, $\rho c_p u_D/(hr_0)$
a_{sf}	specific solid-fluid interfacial surface area (m^{-1})
Bi	Biot number, $hr_0^2/k_{s,eff}$
Pe	Péclet number, $\rho c_p u_D r_0/k_{s,eff}$
T	temperature (K)
r	radial coordinate (m)
φ	circumferential coordinate
u	fluid velocity ($m\ s^{-1}$)
$k_{s,eff}$	effective thermal conductivity of the solid ($W\ m^{-1}\ K^{-1}$)
$k_{f,eff}$	effective thermal conductivity of the fluid ($W\ m^{-1}\ K^{-1}$)
h	specific solid–fluid interfacial heat transfer coefficient ($W\ m^{-3}\ K^{-1}$), $h = h_{sf} \cdot a_{sf}$
h_{sf}	standard solid-fluid interfacial heat transfer coefficient ($W\ m^{-2}\ K^{-1}$)
c_p	specific heat ($J\ kg^{-1}\ K^{-1}$)
z	longitudinal coordinate (m)
k_s	thermal conductivity of the solid ($W\ m^{-1}\ K^{-1}$)
k_f	thermal conductivity of the fluid ($W\ m^{-1}\ K^{-1}$)
r_0	actual radius of the circular tube (m)
R	dimensionless radial coordinate
Z	dimensionless longitudinal coordinate
u_D	Darcian velocity ($m\ s^{-1}$), $u_D = \phi \cdot u_\infty$
u_∞	freestream velocity ($m\ s^{-1}$)

Greek symbols

ρ	fluid density ($kg\ m^{-3}$)
ϕ	porosity
θ	dimensionless temperature
λ	eigenvalue of Eq. (13)
μ	eigenvalue of Eq. (19)
κ	ratio of the effective thermal conductivity of the fluid to that of the solid, $k_{f,eff}/k_{s,eff}$
η	eigenvalue of Eq. (43)
δ	relative error

Subscripts

s	solid phase
f	fluid phase
w	wall

On the other hand, Lu et al. [8] analytically performed the analysis of thermally fully developed forced convective heat transfer characteristics in high porosity open-cell metal-foam filled pipes, and found that the use of metal-foam could dramatically enhance the heat transfer but is at the expense of higher pressure drop. After that, Zhao et al. [9] further investigated the forced convection heat transfer characteristics of high porosity open-cell metal-foam filled tube heat exchangers based on the work of Lu et al. [8]. Hashemi et al. [10] presented an analytical solution of thermally fully developed non-Darcian forced convection gaseous slip-flow in a micro-annulus filled with a porous medium under LTNE condition. Qu et al. [11] proposed an analytical solution of thermally fully developed forced convective heat transfer in an annulus partially filled with metallic foams. Wang et al. [12,13] presented the exact solutions for the forced convection gaseous slip-flow in a porous micro-annulus / micro-tube under LTNE condition. Afterwards, they continued their work to the forced convection in a bidisperse porous medium embedded in a circular pipe and investigated how the bidispersivity affected the thermal characteristics over a wide range of parameter space by analyzing the local Nusselt number and heat transfer performance [14].

It is worth noting that the above studies have provided some valuable information for understanding the forced convection heat transfer in porous media. In these works, the boundary conditions of the temperature field or heat flux are assumed to be axisymmetrical, thus the studied problem is an axisymmetrical problem. However, in engineering practices, if the boundary conditions of the temperature field are asymmetric, then the studied problem shall become a more difficult asymmetric problem. For example, the situation of the circumferential non-uniform wall temperature of heat vacuum glass tubes due to the deviation of the solar radiation angle (Zhong et al. [15], Chang et al. [16]). Cekmer et al. [17] analytically studied the steady, laminar, and fully developed forced convection heat transfer in a parallel plate channel with asymmetric different uniform heat fluxes on the lower and upper walls. Elliott et al. [18] theoretically analyzed the forced convection of a porous channel including internal heat sources with thick, solid walls featuring uneven thicknesses and two types of asymmetric wall thermal boundary conditions. Xu et al. [19] investigated the forced convection in a parallel-plate porous mini/microchannels and the heat fluxes of the upper and the lower walls were supposed to be different. Zheng et al. [20] numerically investigated the enhancement for convection heat transfer of turbulent flow in a solar central receiver tube with a porous medium and non-uniform circumferential heat flux.

To the authors' best knowledge, no efforts have been analytically devoted to evaluate the thermally developing forced convection in a porous circular tube considering the circumferential-dependent entrance temperature under LTNE condition. In view of this, the main objective of the present study is to analytically investigate the thermally developing forced convective heat transfer in a circular tube filled by a porous medium with an asymmetric entrance temperature, and elucidate the dependence of the temperature fields on the relevant parameters.

2. Mathematical model

2.1. Governing equations

The problem under consideration is the thermally developing forced convection heat transfer in a circular tube filled with a porous medium as illustrated in Fig. 1. The fluid flow inside the tube is induced by a constant pressure gradient in the longitudinal direction. A constant temperature T_w is uniformly imposed on the impermeable tube wall, and the circumferential fluid temperature T_{in} at the entrance of the tube is prescribed as an arbitrary function of coordinates r and φ. The following assumptions are made in the present study: (1) The porous medium is homogenous, isotropic, and saturated with the low-conductivity fluid (e.g., air). (2) The flow is steady, incompressible, and obeys Darcy's law. (3) Natural convection, dispersion and radiative heat transfer are ignored. (4) Longitudinal heat conduction is negligible. (5) The flow is hydrodynamically developed but thermally developing with temperature-independent thermophysical properties.

Under the preceding assumptions, the following energy equations under LTNE condition in cylindrical coordinates are expressed as

$$k_{s,eff}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_s}{\partial r}\right)+\frac{1}{r^2}\frac{\partial T_s^2}{\partial \varphi^2}\right)+h(T_f-T_s)=0 \tag{1}$$