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Technical Note

Multiple-relaxation-time lattice Boltzmann model for simulating double-diffusive convection in fluid-saturated porous media

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ABSTRACT

A multiple-relaxation-time (MRT) lattice Boltzmann (LB) model is developed for simulating double-diffusive convection in porous media at the representative elementary volume scale. In the model, the equilibrium moments of the temperature and concentration distributions have been modified, which makes the effective thermal diffusivity and heat capacity ratio as well as the effective mass diffusivity and porosity decoupled. Numerical tests demonstrate that the present model can serve as an accurate numerical method for simulating double-diffusive convection in porous media.

1. Introduction

Double-diffusive convection in fluid-saturated porous media has attracted a great deal of attention because it is a common phenomenon in nature, and it is also frequently encountered in a wide variety of engineering applications, such as geophysical systems, the migration of moisture contained in fibrous insulation, the transport of contaminants in groundwater, the underground disposal of nuclear wastes [1–7]. Over the last several decades, double-diffusive convection in porous media has been studied numerically by many researchers. Conventional numerical methods, such as the finite-element method (FEM) [1,2], the finite-volume method (FVM) [3,4], and the finite-difference method (FDM) [5], have been employed to study double-diffusive convection problems in porous media. Comprehensive reviews of the subject have been given by Ingham and Pop [6], and Nield and Bejan [7].

The lattice Boltzmann (LB) method, as a mesoscopic numerical technique originated from the lattice-gas automata (LGA) method [8], has achieved great success in simulating complex fluid flows and modeling complex physics in fluids [9–16]. With its roots in mesoscopic gas kinetic theory, the LB method exhibits some attractive advantages, such as clear physical pictures, inherently parallel nature, simple algorithm, and easy implementation [9]. Recently, the LB method has also been successfully applied to study double-diffusive convection problems in porous media. Xu et al. [17] investigated double-diffusive convection around a heated cylinder in a square cavity filled with a porous medium at the representative elementary volume (REV) scale. Xu et al.'s study was based on the Brinkman-extended Darcy model. In the literature [18], Chen et al. have developed an LB model for simulating double-diffusive convection in fluid-saturated porous media at the REV scale. Chen et al.'s model was developed based on the generalized non-Darcy model, and by introducing a reference porosity into the equilibrium concentration distribution, it can work well not only for uniform porous media but also for non-uniform porous media.

Up to now, although some progress has been made in studying double-diffusive convection in porous media at the REV scale, some drawbacks of the LB method are apparent. For instance, the effective thermal diffusivity and (reference) heat capacity ratio as well as the effective mass diffusivity and (reference) porosity is coupled. The artificial couplings may do harm to the accuracy of the LB method. Therefore, the objective of this work is to propose an LB model for simulating double-diffusive convection in porous media, in which the effective thermal diffusivity and heat capacity ratio as well as the effective mass diffusivity and porosity is decoupled. Considering that the multiple-relaxation-time (MRT) collision model [19,20] is superior to its Bhatnagar-Gross-Krook (BGK) counterpart [21] in terms of accuracy and numerical stability, the MRT collision model is employed in this work.

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2. Macroscopic governing equations

Based on the generalized non-Darcy model [22–24], the macroscopic governing equations for double-diffusive convection in porous media at the REV scale can be written as follows [1–3,7]:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\left(\frac{\mathbf{u}}{\phi}\right) = -\frac{1}{\rho_0}\nabla(\phi p) + \upsilon_e \nabla^2 \mathbf{u} + \mathbf{F}, \tag{2}$$

$$\sigma \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla \cdot (\alpha_e \nabla T), \tag{3}$$

$$\phi \frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = \nabla \cdot (D_e \nabla C), \tag{4}$$

where $\mathbf{u}$, p, T, and C are the velocity, pressure, temperature, and concentration, respectively; ρ_0 is the reference density, ϕ is the porosity, υ_e is the effective viscosity, α_e is the effective thermal diffusivity, and D_e is the effective mass diffusivity; $\sigma = \phi + (1-\phi)(\rho_s c_{ps})/(\rho_f c_{pf})$ is the heat capacity ratio, in which ρ_f (ρ_s) and c_{pf} (c_{ps}) are the density and specific heat of the fluid (solid matrix), respectively; $\mathbf{F} = (F_x, F_y)$ is the total body force induced by the porous matrix and other external forces, which can be expressed as [25,26]

$$\mathbf{F} = -\frac{\phi \upsilon}{K}\mathbf{u} - \frac{\phi F_\phi}{\sqrt{K}}|\mathbf{u}|\mathbf{u} + \phi \mathbf{G}, \tag{5}$$

where υ is the viscosity of the fluid (υ is not necessarily the same as υ_e); K and F_ϕ are the permeability and inertial coefficient (Forchheimer coefficient) of the porous medium, respectively. The buoyancy force $\mathbf{G}$ is given by $\mathbf{G} = g[\beta_T(T - T_0) + \beta_C(C - C_0)]\mathbf{j}$, where T_0 and C_0 are the reference temperature and concentration, respectively; β_T and β_C are the thermal and concentration expansion coefficients, respectively; g is the gravitational acceleration, and $\mathbf{j}$ is the unit vector in the y-direction.

The inertial coefficient F_ϕ and permeability K depend on the geometry of the porous media. For flow over a packed bed of particles, according to Ergun's experimental investigations [27], F_ϕ and K can be expressed as [28]

$$F_\phi = \frac{1.75}{\sqrt{150\phi^3}}, \quad K = \frac{\phi^3 d_p^2}{150(1-\phi)^2}, \tag{6}$$

where d_p is the solid particle diameter (or mean pore diameter).

The system governed by Eqs. (1)–(4) is characterized by several dimensionless characteristic parameters: the Prandtl number $Pr = \upsilon/\alpha_e$, the thermal Rayleigh number $Ra_T = g\beta_T \Delta T L^3/(\upsilon \alpha_e)$, the solutal Rayleigh number $Ra_C = g\beta_C \Delta C L^3/(\upsilon D_e)$, the Darcy number $Da = K/L^2$, the viscosity ratio $J = \upsilon_e/\upsilon$, the Reynolds number $Re = LU/\upsilon$, the Lewis number $Le = \alpha_e/D_e$, the Schmidt number $Sc = \upsilon/D_e$, and the buoyancy ratio $N = \beta_C \Delta C/(\beta_T \Delta T)$, where L is the characteristic length, U is the characteristic velocity, ΔT is the temperature difference (characteristic temperature), and ΔC is the concentration difference (characteristic concentration).

3. MRT-LB model for double-diffusive convection in porous media

The flow field governed by Eqs. (1) and (2) can be solved by the MRT-LB model proposed in our previous work [29]. In what follows, we only present the MRT-LB equations for the temperature and concentration fields, which are given by

$$\mathbf{g}(\mathbf{x} + \mathbf{e}\delta_t, t + \delta_t) = \mathbf{g}(\mathbf{x}, t)\mathbf{N}^{-1}\mathbf{\Theta}(\mathbf{n}_g - \mathbf{n}_g^{eq})|_{(\mathbf{x},t)}, \tag{7}$$

$$\mathbf{h}(\mathbf{x} + \mathbf{e}\delta_t, t + \delta_t) = \mathbf{h}(\mathbf{x}, t)\mathbf{N}^{-1}\mathbf{Q}(\mathbf{n}_h - \mathbf{n}_h^{eq})|_{(\mathbf{x},t)}, \tag{8}$$

respectively, where the bold-face symbols ($\mathbf{g}$, $\mathbf{h}$, $\mathbf{n}_g$, and $\mathbf{n}_h$) denote b-dimensional column vectors, e.g., $\mathbf{g} = (g_0, \ldots, g_{b-1})^{\mathrm{T}}$ (b represents the number of discrete velocities); $g_i(\mathbf{x}, t)$ and $h_i(\mathbf{x}, t)$ are the temperature and concentration distribution functions, respectively; $\mathbf{N}$ is the transformation matrix, $\mathbf{\Theta}$ and $\mathbf{Q}$ are relaxation matrices.

The transformation matrix $\mathbf{N}$ linearly maps the discrete distribution functions represented by $\mathbf{g}$ and $\mathbf{h}$ to their moments represented by $\mathbf{n}_g$ and $\mathbf{n}_h$, i.e., $\mathbf{n}_g = \mathbf{N}\mathbf{g}$ and $\mathbf{n}_h = \mathbf{N}\mathbf{h}$. Through the transformation matrix $\mathbf{N}$, the collision processes of the MRT-LB Eqs. (7) and (8) can be executed in moment space $\mathbb{M} = \mathbb{R}^b$, while the streaming processes are still carried out in velocity space $\mathbb{V} = \mathbb{R}^b$. In this work, the D2Q5 lattice is employed without loss of generality. The five discrete velocities $\{\mathbf{e}_i\}$ of the D2Q5 lattice are given by

$$\mathbf{e}_i = \begin{cases} (0,0), & i = 0, \\ (\cos[(i-1)\pi/2], \sin[(i-1)\pi/2])c, & i = 1 \sim 4, \end{cases} \tag{9}$$

where $c = \delta_x/\delta_t$ is the lattice speed with δ_t and δ_x being the discrete time step and lattice spacing, respectively. The lattice speed c is set to be 1 ($\delta_x = \delta_t$) in this work.

For the D2Q5 model, the transformation matrix $\mathbf{N}$ can be chosen as [30]

$$\mathbf{N} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ -4 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix}. \tag{10}$$

Accordingly, the equilibrium moment vectors $\mathbf{n}_g^{eq}$ and $\mathbf{n}_h^{eq}$ are defined as follows

$$\mathbf{n}_g^{eq} = (\sigma T, u_x T, u_y T, -4\sigma T + 5\varpi T, 0)^{\mathrm{T}}, \tag{11}$$

$$\mathbf{n}_h^{eq} = (\phi C, u_x C, u_y C, -4\phi C + 5\varpi C, 0)^{\mathrm{T}}, \tag{12}$$

where $\varpi \in (0,1)$ is a model parameter. The temperature and concentration can be obtained via

$$\sigma T = \sum_{i=0}^{4} g_i, \quad \phi C = \sum_{i=0}^{4} h_i. \tag{13}$$

The equilibrium distributions g_i^{eq} and h_i^{eq} in velocity space are given by

$$g_i^{eq} = \begin{cases} \sigma T - (1-\tilde{w}_0)T, & i = 0, \\ \tilde{w}_i T\left(1 + \frac{\mathbf{e}_i \cdot \mathbf{u}}{c_{sT}^2}\right), & i = 1 \sim 4, \end{cases} \quad h_i^{eq} = \begin{cases} \phi C - (1-\tilde{w}_0)C, & i = 0, \\ \tilde{w}_i C\left(1 + \frac{\mathbf{e}_i \cdot \mathbf{u}}{c_{sT}^2}\right), & i = 1 \sim 4, \end{cases} \tag{14}$$

respectively, where $\{\tilde{w}_i\}$ are the weight coefficients: $\tilde{w}_0 = 1 - \varpi$ and $\tilde{w}_{1\sim4} = \varpi/4$, and $c_{sT} = \sqrt{\varpi/2}$ is the sound speed of the D2Q5 model. From Eq. (14), we can find that the model parameter ϖ should satisfy $\varpi < \sigma$ and $\varpi < \phi$

The relaxation matrices $\mathbf{\Theta}$ and $\mathbf{Q}$ are given by

$$\mathbf{\Theta} = \mathrm{diag}(\zeta_0, \zeta_\alpha, \zeta_\alpha, \zeta_e, \zeta_\varepsilon), \mathbf{Q} = \mathrm{diag}(\eta_0, \eta_D, \eta_D, \eta_e, \eta_\varepsilon). \tag{15}$$

respectively. The effective thermal diffusivity α_e and effective mass diffusivity D_e are defined as

$$\alpha_e = c_{sT}^2\left(\frac{1}{\zeta_\alpha} - \frac{1}{2}\right)\delta_t, \quad D_e = c_{sT}^2\left(\frac{1}{\eta_D} - \frac{1}{2}\right)\delta_t, \tag{16}$$

respectively.