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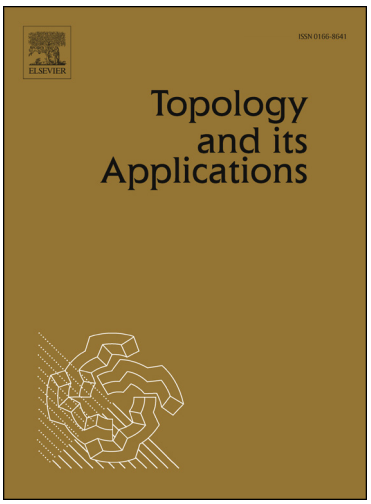

Homeomorphisms of Hashimoto Topologies

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HOMEOMORPHISMS OF HASHIMOTO TOPOLOGIES

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Abstract. We investigate homeomorphisms of different types of Hashimoto topologies based on the Euclidean topology on the real line and classic $\sigma-$ideals.

1. Introduction

Let us start with a notion of Hashimoto topologies introduced independently by Martin in [12] and by Hashimoto in [4].

Let $(X, \mathcal{T})$ be a T_1 topological space and let $\mathcal{I}$ be an ideal of subsets of X, containing all singletons and such that $\mathcal{I} \cap \mathcal{T} = \{\emptyset\}$. We say that such an ideal is admissible. Then the family

$$\{U \setminus P : U \in \mathcal{T}, P \in \mathcal{I}\}$$

is a base of a topology.

Under additional assumptions that $(X, \mathcal{T})$ is a second-countable topological space and $\mathcal{I}$ is a $\sigma-$ideal, the considered family is a topology, denoted by $\mathcal{T}_{\mathcal{I}}$.

This kind of topologies was considered by Lukeš, Malý, Zajíček [11] as 'ideal topologies', by Jankovic and Hamlet [8] and by other authors (Lindner [10], Hejduk [5], Terepeta [14], Bingham and Ostaszewski [3]) as 'Hashimoto topologies'.

Note that such topologies have some common properties.

Theorem 1. *(compare* [12],[4]*) Let $(X, \mathcal{T})$ be a second-countable topological space and let $\mathcal{I}$ be an admissible $\sigma-$ideal. Then*

(1) *$(X, \mathcal{T}_{\mathcal{I}})$ is T_1.*
(2) *The families of connected sets in $(X, \mathcal{T})$ and in $(X, \mathcal{T}_{\mathcal{I}})$ coincide.*
(3) *$(X, \mathcal{T}_{\mathcal{I}})$ does not satisfy the first axiom of countability at any point.*
(4) *$(X, \mathcal{T}_{\mathcal{I}})$ is not regular.*

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