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Determination of equivalent rigidities of cold-formed steel floor systems for vibration analysis, Part II: evaluation of the fundamental frequency

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ABSTRACT

Structural properties of cold-formed steel (CFS) floor systems are essential for evaluating vibration performance of the floor systems. Comprehensive analytical and experimental studies on equivalent rigidities of CFS floor systems for vibration analysis based on orthotropic plate model are presented in this and the companion paper. In Part I of the companion papers, the equivalent rigidities were determined by using Rayleigh's method with considerations of the rotational restraints of joist ends and various configurations of transverse elements. Rotational fixity factors and restraint coefficients were introduced to characterize the effect of the rotational restraints. A joist contribution factor was defined to suitably consider the stiffness contribution of joists for a vibration mode. The Part II of the companion papers presents herein the development of design equations to predict the fundamental frequency of CFS floor systems for vibration serviceability evaluation. Simplified equations were proposed for evaluating the restraint coefficients. More importantly, rotational fixity factors for different CFS framings were investigated based on experimental data. Finally, the predicted results from the proposed method were compared with test results and other methods, and the applicability of the developed equivalent rigidities was assessed.

1. Introduction

As an alternative to traditional wood framing in residential construction, cold-formed steel (CFS) framing inherits many advantages of steel construction. However, if they are not appropriately designed, CFS floors with longer spans, less damping and lighter weight are likely to be susceptible to annoying vibrations induced by human activity such as walking. Design against perceptible vibrations disturbing the floor occupants is referred to as the vibration serviceability in practice. If this issue is not seriously taken into consideration in the design stage for CFS floor systems, there is a risk that the potential market share growth of residential buildings with lightweight structures will diminish [1].

In characterizing the dynamic response of a floor system, the fundamental frequency plays a major role on the evaluation of vibration performance. Building floors are commonly classified into two categories when vibration serviceability is the focus: high- and low-frequency [2]. This classification was originally introduced by Wyatt [3], who suggested that low-frequency floors responded harmonically, with a resonant response, and high-frequency floors acted impulsively with a transient response [4,5]. More specifically, floors with a fundamental frequency less than four times the step frequency will most likely resonate with one of the harmonics, and the resonance will be constantly maintained by subsequent footfalls. On the other hand, when the natural frequency of a floor is above four times the step frequency, the response generated by an individual footfall decays to a comparatively small value by the time the successive footfall begins due to damping. Resonance is thus unlikely to occur, and the vibration will most likely be dominated by a transient response. Hence, the fourth harmonic of the step frequency is commonly used to set the threshold frequency as approximately 10 Hz [6]. Different design methods have been developed for the calculation of resonance and transient responses in floor vibration serviceability, such as Arup's methods by Willford and Young [7]. For lightweight wood floors, Dolan et al. [8] proposed a conservative design criterion which required the fundamental frequency to be greater than 15 Hz for an occupied floor and 14 Hz for an unoccupied floor.

However, the existing design equations may not always provide reasonably accurate prediction for the fundamental frequency of a floor system. Equations of evaluating the fundamental frequency for lightweight floor systems proposed by Dolan et al. [8] and Allen et al. [9] were developed based on the beam model (i.e., one-way system). Although most composite steel floor systems are principally one-way systems and the beam model is simple to be adopted for design practice, stiffness contributions by subfloor, ceiling and transverse elements may

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not be appropriately considered. It is probable that the transverse flexural stiffness associated with transverse elements such as bridging, blocking and strong-back, etc., has traditionally received less attention in design practice due to lack of research on identifying the characteristics of such elements. Furthermore, the boundary conditions of CFS floors are always simplified as simply supported, which is not consistent with actual conditions. The supports of joist ends in practical construction are demonstrated to be elastically restrained against rotation [10–13]. This rotation stiffness has been proven to be influential in the prediction of natural frequencies of wood framed floors [14] and CFS floors [11]. This suggests that actual end support conditions in CFS construction should also be accounted for in vibration serviceability design. In addition, it was seen from the experimental investigation [11] that the presence of transverse elements, such as strongback, and as well as non-structural components, like the ceiling, have made considerable improvements to the vibration performance of CFS floors [11]. These elements need to be considered when evaluating the vibration performance of CFS floors.

Efforts made by Ohlsson [15] and Chui [16] to develop design equations using the equivalent plate theory have had some success on accounting for the contribution of the transverse elements. Both methods are limited to plates simply supported on all edges whereas the boundaries of joist ends in practical construction are known to be elastically restrained against rotation. Ohlsson [15] calculated the frequencies by assuming a simply supported rectangular orthotropic plate for lightweight floor systems but the equivalent structural properties were not provided. Chui [16] presented a rib-stiffened plate model for frequency analysis. This rib-stiffened plate model was proposed by Timoshenko and Woinowsky-Krieger [17], which is effective for closely spaced stiffeners. However, the transverse elements are few and placed far apart; and thus, the rib-stiffened plate model proposed based on lightweight timber floors in [16] may not be applicable to CFS floor systems.

The main purpose of the present paper is to develop the design equations for calculating the fundamental frequency of a CFS floor system in considerations of transverse elements and actual boundary conditions. This paper is the part II of two companion papers of determination of the flexural and torsional rigidities of CFD floor systems. In Part I, the analytical derivation on the equivalent rigidities of CFS floor systems for vibration analysis has been presented [18]. In this paper, by using the equivalent rigidities previously developed, simplified design equations are developed for calculating the fundamental frequency of the floor systems. More importantly, rotational fixity factors for different CFS framings are investigated based on test results. At last, the predicted results from the proposed method are compared with test results, and the applicability of the developed equivalent rigidities are assessed.

2. Evaluation of the fundamental frequency of CFS floor systems

The equivalent rigidities of CFS floor systems as shown in Fig. 1 have been derived in the companion paper [18]. In order to calculate the fundamental frequency, the previously defined factors and coefficients should be determined first. Then, evaluation procedure will be demonstrated in details for calculation of equivalent flexural and torsional rigidities.

2.1. Joist contribution factor, ϵ_j

For the first vibration mode, the joist contribution factor introduced in the companion paper [18] is expressed as

$$\epsilon_j = \sum_{i=1}^{N_j} \phi^2(y_i) = \sum_{i=1}^{N_j} \left(\sin \frac{\pi y_i}{b}\right)^2 \tag{1}$$

where N_j is the number of joists of the floor, y_i is the location of the ith

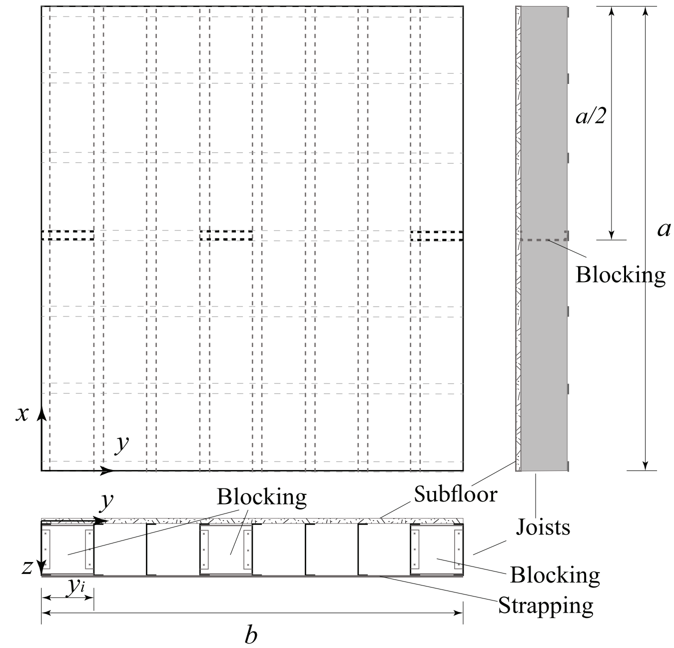

Fig. 1. Layout of a typical CFS floor system.

joist, and b is the floor width. Since

$$\sum_{i=1}^{N_j} \left(\sin \frac{\pi y_i}{b}\right)^2 = \sum_{i=1}^{N_j} \sin^2\left(\frac{\pi(i-1)s}{(N_j-1)s}\right) = \sum_{i=1}^{N_j} \sin^2\left(\frac{\pi(i-1)}{N_j-1}\right) = \frac{N_j-1}{2} \tag{2}$$

in which $s = b/(N_j - 1)$ is the space of adjacent joists, the joist contribution factor for the evaluation of the fundamental frequency is

$$\epsilon_j = \frac{N_j-1}{2} \tag{3}$$

2.2. Simplified equations of restraint coefficients

In the companion paper [18], restraint coefficients c_1, c_2 and were defined in Eqs. (33a), (33b) and (41a) to consider the effect of the rotational restraints of the joist ends on the rigidities, which involves some infinite series and can be determined for each mode shape by truncating the infinite series to be finite terms. In attempting to compute restraint coefficients for engineering practice, simplified formulas were obtained by the polynomial curve fitting to numerical results (infinite series were truncated at $m = 1000$). Then, the coefficients can be conveniently approximated by

$$c_i = Ar^3 + Br^2 + Cr + 1, \quad i = 1, 2, \text{ and } 3 \tag{4}$$

where the rotational fixity factor at joist end x_0 and x_a: $r = r_{x0} = r_{xa}$; and A, B and C are constants as shown in Table 1. The coefficient of determination R^2 [19] is used to measure the closeness between the simplified formula and the numerical results. Values of R^2 change within the range from 0 to 1. An R^2 of 1 indicates that the predictions perfectly fit the data. Table 1 shows that all the three values of R^2 are

Table 1
Constant values of simple formulas for restraint coefficients.

Coefficients	A	B	C	R^2
c_1	4.619	− 2.277	1.757	0.9994
c_2	0.662	− 0.522	0.097	0.9929
c_3	0.284	− 0.134	0.109	0.9995

Note: R^2–Coefficient of determination.