Thin-Walled Structures 132 (2018) 25–35

Contents lists available at ScienceDirect

Thin-Walled Structures

journal homepage: www.elsevier.com/locate/tws

Full length article

Determination of equivalent rigidities of cold-formed steel floor systems for vibration analysis, Part I: Theory

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ARTICLE INFO

Keywords:
Equivalent orthotropic rigidities
Cold-formed steel floor systems
Rotational restraints
Rayleigh's method
Rotational fixity factor
Restraint coefficient

ABSTRACT

Cold-formed steel (CFS) floor systems have been increasingly used in residential and commercial buildings. The evaluation of vibration performance of these floors is a great challenge to be dealt with in the design stage. One of the reasons is that there is a lack of an applicable procedure to accurately determine the structural properties of the floors for vibration serviceability evaluation. Current design approaches for floor vibration serviceability are based on simplified equivalent models which lack the capability to account for effects of rotationally restrained floor edges and transverse elements. In this study, comprehensive analytical and experimental investigations have been carried out on the elastic rigidities of CFS floors for vibration analysis based on equivalent orthotropic plate model and results are reported in the present paper and its companion paper. In the present paper, the equivalent rigidities were determined by using Rayleigh's method with considerations of the rotational restraints on the joist ends and different transverse elements. Rotational fixity factors were introduced to represent the rotational restraints and restraint coefficients were designated for frequency calculation. A joist contribution factor was defined to suitably consider the stiffness contribution of joists for a vibration mode. Upon validating the proposed flexural rigidities for vibration analysis, a design equation was developed in the companion paper for predicting the fundamental frequency of the floor system for vibration serviceability evaluation. The results obtained from the equation were compared to those of tests. In addition, in the companion paper, rotational fixity factors for different type of CFS framings were discussed for evaluating the fundamental frequency based on the existing experimental data.

1. Introduction

Cold-formed steel (CFS) floor systems, as shown in Fig. 1, generally comprise a series of equally spaced CFS joists sheathed with subfloors and reinforced by transverse elements. The subfloor could be plywood, oriented strand board (OSB), cementitious board, or corrugated steel deck with lightweight gypsum-based underlayment. Typical transverse elements including blocking, strongback and bridging provide required transverse floor stiffness and lateral bracing of CFS joists. In the past few decades, CFS floor systems have been increasingly used in residential construction and other lightweight frame construction in North America as a cost-effective alternative to traditional wood-framed floors. Compared to this counterpart, CFS floors are relatively lighter in weight, have less damping and allow for longer spans. As a result, CFS floors may become prone to vibration induced by external sources such as human activities. Therefore, the floor vibration serviceability needs to be appropriately addressed in the design.

Prior to the evaluation of vibration performance of CFS floors, the structural properties such as floor flexural and torsional rigidities need to be obtained for vibration analysis. Generally, CFS floor systems can be regarded as rib-stiffened plates in which a thin plate (i.e., subfloor) is eccentrically reinforced by a series of equidistant stiffeners (i.e., CFS joists) in the longitudinal direction and one to three rows of stiffening elements (i.e., Blockings, Strappings and Strongback) in the transverse direction. The dynamic behavior of stiffened plates has been studied for more than half a decade [1–5]. Various analytical approaches and numerical methods have been proposed to evaluate the dynamic characteristics of the plate-stiffener system [6–17]. However, CFS floor systems are irregularly stiffened plates in which floor joists are closely and evenly spaced with few irregularly spaced transverse elements. In the direction of floor joists, the effective rigidity constants of the equivalent plate can be determined by using so-called "smeared-out" technique, provided that ratios of joist spacing to plate dimensions are small enough to insure approximate homogeneity of stiffness such that the orthotropic plate model is applicable [6]. In contrast, this continuum idealization is not suitable for the transverse elements which

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https://doi.org/10.1016/j.tws.2018.08.001
Received 29 December 2017; Received in revised form 25 June 2018; Accepted 1 August 2018

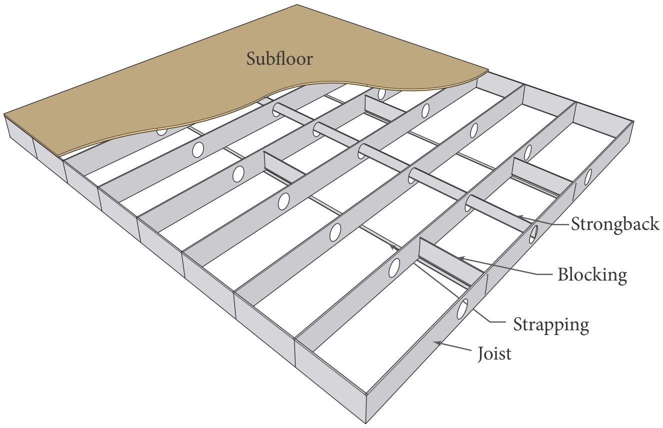

Fig. 1. Typical CFS floor systems.

should be considered discretely because they are few and far apart. Furthermore, the joist ends of CFS floors are more likely rotationally restrained in practice instead of simply supported. As a result, the vibration analysis can become more complicated. This has motivated the search for an approximate technique for determining the equivalent flexural and torsional rigidities for daily engineering practice without recourse to complex analytical methods or time-consuming numerical solutions.

Presented in this paper and its companion paper are a comprehensive analytical and experimental investigation of elastic rigidities for vibration analysis of CFS floor systems. The present paper provides an analytical approach for the determination of the equivalent rigidities. The approach is based on Rayleigh's method and the rotational restraints at joist ends are considered. In addition, contributions made by different transverse elements on the rigidities were also taken into account. Rotational fixity factors were introduced to represent the rotational restraints and restraint coefficients were designated for frequency calculation. A joist contribution factor was subsequently defined to consider the stiffness contribution of joists for different mode shapes. Effects of different arrangements of transverse elements on the equivalent rigidities were investigated. In particular, the issues of torsional rigidity of joists, discrete transverse elements, effects of two types of ceilings and free edges were discussed.

2. Methodology

For the purpose of design, rib-stiffened plates can be analyzed by equivalent orthotropic plates. Such an approach is often sufficiently accurate and usually less complicated than an approach that considers the stiffeners discretely [18]. With the determination of the equivalent rigidities, the equivalent orthotropic plates can be used to simulate the desired behavior of original CFS floor systems. For vibration analysis, the equivalent orthotropic plates are enable to provide accurate solutions on natural frequencies and modal shapes of original floor systems [6,9]. Previous efforts have been devoted to obtaining the rigidities of an equivalent orthotropic plate [6,7,9,1]. Nevertheless, most existing methods were developed in the context of static problems, such as those in [6,7]. As mentioned in [9], the situation in the static case is simple because the deflection pattern of a plate is preformed in the shape of its first mode shape. As shown in Fig. 2(a), there is no nodal lines presented in the first mode shape. In contrast, the issue of nodal lines in higher mode shapes should be considered in vibration analysis. In the case of higher modes as shown in Fig. 2(b) and (c), if a nodal line lies at the location of a stiffener, the corresponding stiffener would not have influence on vibration behavior of the plate in that particular mode [9].

Iyengar and Iyengar [9] employed the Rayleigh's method to determine the equivalent rigidities of bidirectionally stiffened plates in free vibration. In their study, by applying eigenfunctions of beams with the same boundary conditions of plates, the natural frequencies for the stiffened and the equivalent orthotropic plate were obtained. The equivalent rigidities were determined through equating the natural frequencies of the stiffened plates with those of the equivalent orthotropic plates. A similar method was also proposed by Smith and Chui [19] to predict natural frequencies of lightweight wood floors.

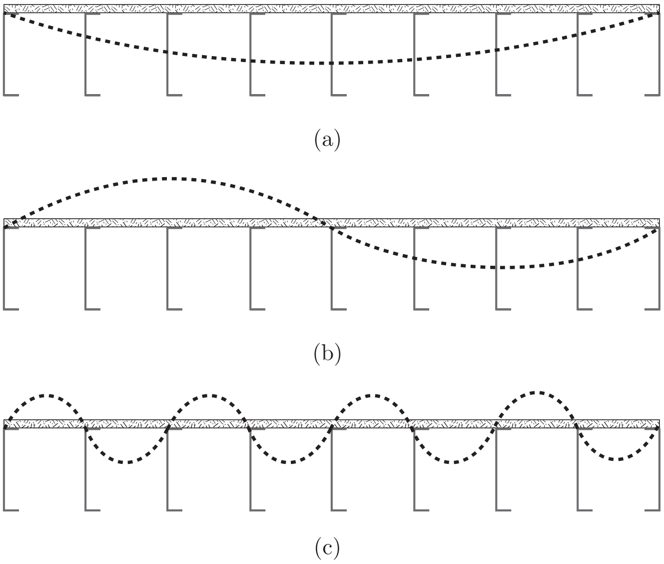

Fig. 2. Front sectional views of CFS floors and mode shapes (i.e., dotted lines).

In the present paper, the Rayleigh's method is also applied to obtain equivalent rigidities of a CFS floor system. It is well known that more accurate results can be obtained by using the Rayleigh-Ritz method with a series of admissible functions because the results associated with the Rayleigh's method achieve only a first approximation to a vibration frequency by using a single admissible function [20]. However, the application of the Rayleigh-Ritz method can be computational intensive because the plate deflection shape $W(x, y)$ is expressed as the sum of a series of products of undetermined weighting coefficients and admissible functions. The single term approximation of the Rayleigh's method was adopted by Warburton [21] to derive approximate expressions for the frequencies of isotropic plates with various boundary conditions, such procedure was subsequently applied to orthotropic plates by Hearmon [22]. The accuracy of the procedure was assessed and validated by comparison with published analytical and experimental results [23].

In the context of the engineering practice, the Rayleigh's method is adopted in this research to establish the equivalent rigidities of CFS floor systems. The selection of appropriate admissible functions is critical for achieving desired accuracy in calculating natural frequencies of floors using the Rayleigh's method. Characteristic functions for beams with the same boundary conditions of plates are often adopted in practice as admissible functions. Thus, to facilitate the discussion on modelling CFS floor systems by using the equivalent orthotropic plates with rotationally restrained edges, the free vibration of rotationally restrained beams (i.e., simply supported beams with ends elastically restrained against rotation) is investigated firstly.

3. Free vibration of rotationally restrained beams

In standard textbooks of vibration such as [24], the solutions of free vibration of beams can be assumed as combined hyperbolic functions and trigonometric functions in which the unknown constants can be subsequently determined by applying the boundary conditions. For beam with complex boundary conditions, such method might not be efficient. Recently, the method of finite integral transform [25–27] has been successfully applied to obtain exact series solutions of rotationally