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Tree inference: Selective influence in multinomial processing trees with supplementary measures such as response time

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HIGHLIGHTS

- Multinomial Processing Trees are successful models of many phenomena.
- Typically only response probability is modeled. Response time can be modeled as well.
- Selective influence of factors can be tested with response probability and time.

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ABSTRACT

Multinomial Processing Trees are successful models of response probabilities for many phenomena. Empirical validation is often based on manipulating an experimental factor intended to selectively influence a process represented in a Multinomial Processing Tree, to see whether the factor indeed has an effect on and only on a parameter associated with that process. Response times are rarely included, but have great potential for increasing resolution. We consider Multinomial Processing Trees in which outcomes of processes represented by vertices occur with probabilities (as usual), and also take time. For response time itself, the method of selectively influencing processes is well developed. Established tests are based on response time means and distribution functions. We modify well established tests so they can be applied to Multinomial Processing Trees in which responses fall into two classes, say, correct and incorrect. The new tests are based on response time means and distribution functions, each multiplied by response probability. If two experimental factors selectively influence two different vertices in a two class Multinomial Processing Tree, the tree is equivalent to one of two simple trees. Patterns in response probabilities and times will indicate which of the two trees accounts for the data. In one of the two trees, the selectively influenced vertices are executed in order, in the other they are not. If there are more than two response classes, each class can be tested separately. If the patterns do not occur, no Multinomial Processing Tree exists in which the two experimental factors selectively influence two different vertices. We demonstrate the method with simulated data from a two factor experiment.

Analysis of response time has a long history, (e.g., Helmholtz, 1883) as does analysis of response accuracy (e.g., Ebbinghaus, 1885/1913). Most analyses focus on one variable, not from belief that the other is unimportant but from lack of techniques for analyzing them in a unified way. A paper often reports an Analysis of Variance on response time, followed by another Analysis of Variance on proportion of correct responses, followed by discussion of where the separate analyses agree and disagree. An exception is work on certain mental processes, decision and search in particular, for which well-developed theories explain the duration of the process and the outcome of the process in a unified way (see, e.g., Ratcliff, 1978; Smith, 2000; Vickers, 1979). Theories about a single process often assume the other processes required in a task simply precede or follow the single process, but for many tasks such a serial arrangement is an oversimplification. There is a need for models that allow complex arrangement of individual processes yet treat response time and response class in a unified way. Multinomial Processing Trees are a promising foundation, already well established as models of response accuracy.

Multinomial Processing Trees are successful models in many realms, including perception, memory, and social cognition. For reviews see Batchelder and Reifer (1999), Erdfelder et al. (2009), and Hütter and Klauer (2016). Modeling is usually of probabilities that responses fall into various classes, such as correct and incorrect. A few investigators, including Link (1982), Hu (2001) and, recently, Heck and Erdfelder (2016, 2017) and Klauer and

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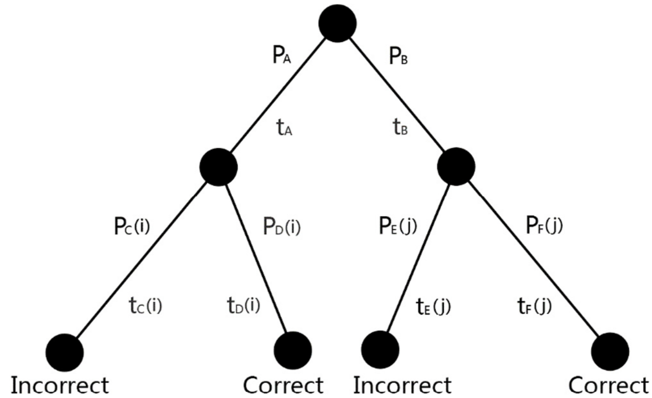

Fig. 1. A Multinomial Processing Tree, the Standard Tree for Unordered Processes. A vertex represents a process. An arc L descending from a vertex represents an outcome of the process. The outcome occurs with probability π_L and takes time t_L. The probability and time for arc C depend on the level i of a factor. The probability and time for arc E depend on the level j of another factor. Other notation is similar.

Kellen (2018) have extended Multinomial Processing Trees to response time. A potentially informative feature of such models is that they assume a person can perform a task in more than one way, so they are capable of explaining nonmonotonic relations between response time and accuracy (Schweickert & Zheng, 2018a). Many well developed techniques for response time analysis would be useful, if modified to apply to Multinomial Processing Trees. Here we present some such modifications, based on selectively influencing processes with experimental factors.

An experimental factor selectively influences a process if changing the level of the factor changes parameters unique to that process, leaving all else invariant. Sternberg (1969) pioneered selective influence for response times. It is now often used for response probability in Multinomial Processing Tree (MPT) models. For example, in an immediate recall experiment, Chechile (1977) found that changing the phonological similarity of items changed a storage parameter in an MPT, leaving other parameters invariant. As Batchelder and Alexander (2013, p. 1209) say, "In almost all of the articles proposing a new MPT model are selective influence studies designed to validate the interpretation of the parameter estimates". For a survey of selective influence for response probabilities, see Schweickert, Fisher, and Sung (2012). For a general conception, see Dzhafarov (2003).

In a Multinomial Processing Tree (Fig. 1), a vertex represents a process such as memory retrieval. Schweickert and Chen (2008) and Schweickert and Xi (2011) developed response probability tests of whether two factors selectively influence two different processes, each represented by a different vertex in an arbitrary Multinomial Processing Tree. Here we extend the tests to response times and other measures for the case in which responses fall into two classes, e.g., correct and incorrect. The tests are variations of established tests for response time (e.g., Dzhafarov, Schweickert, & Sung, 2004; Houpt, Blaha, McIntire, Havig, & Townsend, 2014; Roberts & Sternberg, 1993; Schweickert, 1978; Sternberg, 1969; Townsend & Nozawa, 1995). We investigate the feasibility of the tests with simulations.

A two class Multinomial Processing Tree is one in which responses fall into two classes. Two factors selectively influencing different vertices in such a Multinomial Processing Tree are very informative. The MPT is equivalent, as we will explain, to one of two relatively simple trees, so the investigator need only consider these two. If the two vertices are in order in the MPT, the order can sometimes be determined from the data. Notably, if the tests fail no two class Multinomial Processing Tree is possible in which the two factors selectively influence two different vertices. The investigator can quickly learn another type of model is needed.

If there are more than two response classes, every class can be tested separately. It is necessary that the separate tests are satisfied for every response class if a single MPT with a terminal vertex for each response class is able to account for the data, with each of the two factors selectively influencing a different vertex (Schweickert & Zheng, 2017). We emphasize that if the tests fail, an MPT may account for the data, but not an MPT in which each of the two factors selectively influences a different vertex.

1. Probability in multinomial processing trees

A Multinomial Processing Tree (Fig. 1) consists of points, called *vertices*, joined by lines, called *arcs*. Our introduction here is casual, for a more formal description see Purdy and Batchelder (2009). Each vertex represents a mental process, such as perception or memory retrieval. An arc descending from a vertex represents an outcome of the process, such as successful memory retrieval. When an event such as stimulus presentation occurs, the first process begins. In the figure this process is represented by the vertex at the top of the tree, called the *source*. On a particular trial, a single outcome of the first process occurs, the arc representing the outcome is traversed, and the vertex at the end of the arc is reached. This vertex represents a further process, which is executed. Such steps continue until a vertex is reached that has no descending arcs. At such a *terminal vertex* a response is made. The steps form a path from the source to the terminal vertex. (Some authors say a vertex represents a state, and an edge or entire path represents a process. We do not.) Responses fall into various classes, such as correct and incorrect. A particular terminal vertex is associated with only one class of response.

When a vertex is reached, each arc descending from it has a probability of being traversed. In Fig. 1, for example, the probability arc A is traversed is denoted π_A. The probability of traversing all the arcs on a path from the source to a terminal vertex is the product of the probabilities on the arcs. Each terminal vertex has a probability of being reached on a path from the source. The probability a response falls into a particular class is the sum of the probabilities of the terminal vertices associated with the class.

2. Supplementary measures in multinomial processing trees

Some processing trees include a measure in addition to probability. In a decision tree, each arc has a probability of being selected and a gain or loss is obtained. The net gain of a path is the sum of the gains and losses associated with the arcs on the path. In another example, Rosenbaum (1980) proposed a processing tree for the task of reaching straight ahead to a stimulus a certain distance away. At each vertex, an outcome is selected with a certain probability and the outcome adds or subtracts an amount to the distance goal. The distance reached equals the sum of the amounts in the distance goal.

Some examples incorporate time. Rosenbaum, Kenny, and Derr (1983) proposed a processing tree for a movement sequence, such as pressing keys on a piano. A vertex leads with a certain probability to the correct next vertex, or with a different probability to an incorrect next vertex. Executing the process represented by a vertex requires a certain amount of time. The time between two successive movements is the sum of the times required by the vertices on the path from one movement to the other.

Similarly, in Hu's (2001) model when processing is carried out at a vertex, each arc descending from the vertex has a probability of being selected and takes time. Using the assumption that probabilities on a path multiply and times on a path add, Hu derived