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A framework for identifying mathematical arguments as supported claims created in day-to-day classroom interactions

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ABSTRACT

This article addresses how to distinguish mathematical arguments created during whole class discussions in grades 3–5 in Sweden, while taking a broad range of communicational resources, such as speech, drawings and symbols, into account. We present a step-by-step framework of how to systematically reconstruct mathematical arguments. The framework is developed drawing on Toulmin's model of argumentation and a multimodal approach. When giving account for the framework, we show how various communicational resources convey the mathematical meaning of the arguments created. The framework can be used for further research investigating interaction in classroom settings, for teacher students as a basis for reflection during practicum periods, as well as a lens for teachers in identifying informal and formal mathematical arguments in day-to-day communication in the mathematics classroom.

1. Introduction

Communication and the language used in mathematics classroom have attracted a lot of attention within mathematics education for some time (Morgan, Craig, Shuette, & Wagner, 2014), as a result of the acknowledgement of the social aspects of teaching and student learning (Lerman, 2000). As part of mathematics classroom communication, the interest in arguments is noticeable in many studies (e.g., Krummheuer, 2007; Mueller, 2009). As we will address below, there is a vast body of literature on mathematical arguments, often drawing on a model by Toulmin (2003). Less attention has been paid to detailed accounts of *how* to distinguish and reconstruct mathematical argument in the analytical process. This 'how-question' is the focus of this article. In order not to exclude analysis of informal arguments as part of day-to-day classroom interaction, we have adopted a multimodal approach (Hodge & Kress, 1988; Kress, 2010; Van Leeuwen, 2005), acknowledging a broad range of communicational resources (modes), such as speech, symbols, and drawings. The aim of this study was to develop a framework for identifying mathematical arguments as part of day-to-day classroom communication. Our two research questions specify our aim:

- Research question 1 (RQ1): How can mathematical arguments be identified in the sense of supported claims?
- Research question 2 (RQ2): How can mathematical arguments be identified when conveyed through a broad range of modes (either separately or at the same time)?

The arguments in the study were created during discussions where one or several individuals (teacher and/or students) were actively justifying mathematical claims and conclusions for others to pay attention to. We give account for a step by step framework, used for the analysis of a rather large data set, where supported claims were identified. We also illuminate how arguments that might

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easily be missed out on could be captured through the utilisation of a multimodal approach. The resulting framework, presented in this article, may offer a basis for researchers, university students, and teachers who want to pursue analysis of mathematical arguments as part of the interactions in mathematics education activities. Moreover, this study can be seen as making a contribution on a theoretical as well as a methodological level, as it shows how one can analyse and gain deeper insights into how arguments are communicated during day-to-day interactions, in relation to the teaching and learning content they address.

2. Research overview

Arguments are often studied in connection to argumentation and/or reasoning. In this overview, we take into account research with relevance for a better understanding of how mathematical arguments as part of classroom communication have been identified and studied. As a consequence, we draw not only on research on arguments but also on research on argumentation and reasoning, paying specific attention to how arguments, argumentation and reasoning in mathematics have been identified and analysed.

2.1. Identifying arguments, argumentation, and reasoning

Stylianides (2007) connects to the process of identifying arguments in data, when he describes an argument as “a connected sequence of assertions intended to verify or refute a mathematical claim” (p. 2). This is similar to Toulmin, Rieke, and Janik, (1979) and their description of an argument as the sequence of interlinked claims and the reasons connecting them, also understood as “train of reasoning”. Here reasoning is understood as the central activity of presenting reasons in support of a claim. Arguments as interlinked claims were investigated by Knipping and Reid (2015) who describe complex arguments in the proving process in secondary mathematics classrooms. Within these arguments, they identified each separate argument with its interlinked claim as a “local argument”. In this article, we view each supported claim as an argument (the claim included), interlinked with other arguments, or not.

In the presentations of reasons, an argument for a claim/conclusion can be viewed as part of a mathematical argumentation (Krummheuer, 1995), where the questioning of others’ arguments, criticizing reasons, and so forth are included (Toulmin et al., 1979). Argumentation may also be described as incorporating a back and forth flow of contributions by individuals interpreting the meaning of another’s statement (as part of an argument) and adjusting their own as a result (Rasmussen & Stephan, 2008), more in the manner of collaborative argumentation (Coffin & O’Halloran, 2008).

To present reasons, as part of an argument, to reach a conclusion, can also be viewed as part of the process of reasoning, often in connection to problem solving, as the way of reaching a conclusion (Lithner, 2008; see also Mueller, 2009; Yankelewitz, Mueller, & Maher, 2010). Lithner describes this as a “sequence of reasoning” (p. 257). This can also be done in a collaborative way (Mueller, Yankelewitz, & Maher, 2012).

In order to identify the arguments in data, many studies refer to a model of argumentation by Toulmin (e.g. 2003) (Krummheuer, 1995, 2007 see also, e.g., Arzarello & Sabena, 2011; Conner, Singletary, Smith, Wagner, & Francisco, 2014; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; González & Herbst, 2013; Inglis, Mejia-Ramos, & Simpson, 2007; Kosko, 2016; Lavy, 2006; Yankelewitz et al., 2010). By using the model (see Fig. 1), the structure of an argument can be mapped. Krummheuer (1995) introduced a reduced model, and even though the limitations of using the reduced model have been addressed by Inglis et al. (2007), especially when investigating arguments similar to those by mathematicians, it can be viewed as sufficient to analyse argumentation/arguments at the school level (Knipping & Reid, 2015; Krummheuer, 1995). Many studies within mathematics education have followed the work by Krummheuer (1995), and thereby explicitly or indirectly used the model, as is the case in this article.

2.2. Methods of data collection

In order to capture arguments during lessons in mathematics, different methods for data collection are used, depending on the interest of the study. The main methods identified are written work by students and/or video recording (often along with other

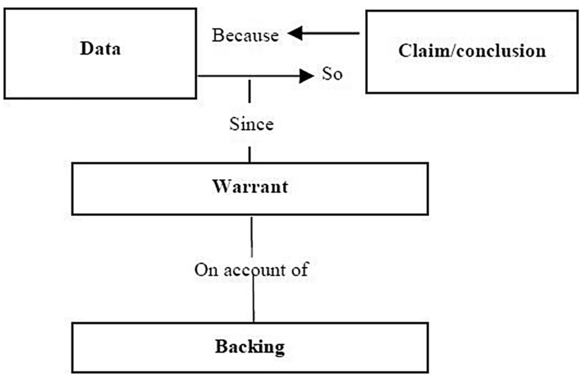

Fig. 1. A model of argumentation (Toulmin, 2003; see also Krummheuer, 1995).