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An international perspective on knowledge in teaching mathematics

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ABSTRACT

This article provides an overview of the Special Issue of the *Journal of Mathematical Behavior*: An international perspective on the knowledge in teaching mathematics, with a focus on the primary and elementary levels. The work originated with presentations given at the *International Congress on Mathematics Education* (ICME) 13 in Hamburg, Germany, in the Summer of 2016. The papers were solicited from the participants in *Topic Study Group* (TSG) 45: *Knowledge In/For Teaching Mathematics at Primary Level*. A primary goal for TSG 45 was to advance theories about the teacher content knowledge and pedagogical content knowledge required for focusing instruction on building mathematical understanding and promoting student reasoning. This Special Issue emphasizes the role that teachers play in supporting that development. A significant corpus of research has identified major factors influencing how students build mathematical ideas and construct pathways of reasoning. However, our current understanding is that what is lacking is a conceptual framework that identifies and integrates factors regarding teachers' roles in supporting students' mathematical learning; and also, identifying the set of conditions and resources that optimally support that learning. We address this area of inquiry by organizing this overview article into the following sections: Cognitive perspectives; pedagogical perspectives; attending to students' reasoning; language, communication and dialogue perspectives; and finally, we conclude with research-based implications for teaching practice and teacher professional learning.

1. Introduction

This article provides an overview of the Special Issue of the Journal of Mathematical Behavior, *An International Perspective on Teaching Mathematics* with a focus on teacher education at the primary and elementary levels. The work originated with presentations delivered at the *International Congress on Mathematics Education* (ICME) 13 in Hamburg, Germany, in the Summer of 2016. The papers were solicited from the participants in *Topic Study Group* (TSG) 45: *Knowledge In/For Teaching Mathematics at Primary Level*. A primary goal for TSG 45 was to advance theories about the teacher content knowledge and pedagogical content knowledge needed for focusing instruction on building mathematical understanding and promoting student reasoning. In particular, the focus of the Special Issue is on the role that teachers play in supporting that development. A significant corpus of research has identified major factors influencing how students build mathematical ideas and construct pathways of reasoning. However, our current understanding is that

Within these perspectives, the contributions of TSG 45 papers are assembled in this special issue.

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what is lacking is a conceptual framework that identifies and integrates factors regarding teachers' roles in supporting students' mathematical learning and also the set of conditions and resources that optimally support learning. We address this area of inquiry by organizing this overview article into the following sections: Cognitive perspectives; pedagogical perspectives; attending to students' reasoning; language, communication and dialogue perspectives; and finally, we conclude with research-based implications for teaching practice and teacher professional learning.

2. Cognitive perspective

Fundamental to our perspective is a belief that learning mathematics is based on students making sense of mathematical concepts and procedures. Learning mathematics requires students to cycle through phases of building representations, retrieving and or constructing relevant knowledge, mapping representations to knowledge, and using these as a basis for action toward problem solving including communication (Davis & Maher, 1990). To give an example, there is compelling evidence that young children, prior to formal instruction, make use of the idea of mathematical proof in justifying solutions to problems. Children verbally articulate arguments in the form of: proof by cases; induction; upper/lower bound and contradiction (Maher & Martino, 1998; Maher & Davis, 1995; Maher & Martino, 1996; Maher & Yankelewitz, 2017). Students' justifications may be motivated by their efforts to make sense of the problem, notice patterns, and pose theories (Maher & Martino, 2000; Mueller, Yankelewitz, & Maher, 2011). Furthermore, students refine their solutions through discussions, as they negotiate meaning with other students and structure their own investigations (Maher, 2005; Weber, Maher, Powell, & Lee, 2008). Research has established that students' ability to provide convincing mathematical justifications assists them in their own understandings of mathematical proof and also serves as a resource for them to validate mathematical statements (Yackel & Hanna, 2003). Research shows that middle school students rely on their sense-making and reasoning to provide convincing arguments for their solutions to problems (Mueller & Maher, 2010a; Mueller & Maher, 2010b; Mueller & Maher, 2009; Mueller et al., 2011).

Informal mathematics, which includes the construction of personally meaningful representations, also plays a significant role in the cognitive process by which students link their innate curiosity with their natural inclinations to search out patterns and relationships (Baroody & Ginsburg, 1990). Thus, informal mathematics provides the opportunity for students to assimilate formal mathematics as a gradual process of shifting from concrete, informal and personally meaningful knowledge to more abstract, formal and systematic knowledge (Baroody & Ginsburg, 1990). Students relate partially familiar knowledge to what they already know. However, knowledge that is unfamiliar can be disregarded by students, because there is no way to connect and potentially integrate that knowledge into their existing schema.

In a similar manner, communication and use of representations are fundamental to students' building mathematical cognition when it is viewed as "an act of sense-making that is socially constructed and socially transmitted" (Schoenfeld, 1992, p. 339). Mathematical situations are communicated through statements of problems or tasks, and students use this information to construct their mental representations (Davis & Maher, 1990; Davis, 1984). Students express and refine their mental representations by creating external representations in the forms of language (both oral and written), drawings, symbols, written texts, among other forms of representation that can be communicated to others; this process, in turn has the potential to interact and impact students' learning.

Classrooms that function as communities of practice in which social values and norms for behavior are transmitted, can shape perceptions and actions to mediate the learning experience (Lave & Wenger, 1991). These include the norms for communicating mathematical ideas, socio-mathematical norms (Yackel & Cobb, 1996) and values associated with reasoning and justification (Francisco & Maher, 2005; Maher et al., 2010; Palius & Maher, 2011). The ways in which students discuss mathematical ideas and behaviors are consequential to the mathematics they attempt to learn, since learning environments both allow for and are constituted by the discourses that occur within them (Edelsky, Smith, & Wolfe, 2002). A key aspect of enacting a cognitive perspective in classrooms is building multi-dimensional knowledge that is idiosyncratic to the teaching of mathematics. Building on the work of Shulman (1986); Hill, Ball, and Schilling, 2008 proposed two categories of knowledge: subject matter knowledge and pedagogical content knowledge, expanding on the knowledge of content and students. Mathematics subject matter knowledge comprises three sub-categories: common content knowledge; specialized content knowledge; and knowledge at the horizon. Pedagogical content knowledge includes: knowledge of content and teaching; knowledge of content and students; and knowledge of curriculum.

3. Pedagogical perspective

It is well accepted that teaching is a challenging and complex activity. For mathematics, the task design and the implementation procedures are central concerns in establishing optimal circumstances for students' learning (Francisco and Maher, 2005; Henningsen & Stein, 1997; Mueller et al., 2011; Stein & Lane, 1996; Sullivan, Mousley, & Jorgensen, 2009a; Sullivan, Mousley, & Jorgensen, 2009b; Sullivan et al., 2014). Across all ages and contexts, formal and informal, certain tasks tend to elicit certain forms of reasoning when students are required to provide a justification for their solutions (Francisco & Maher, 2005; Yankelewitz, Mueller, & Maher, 2010). Inviting students to convince themselves and others about their solutions and explain why the solutions are correct and complete is a highly effective practice in stimulating students' learning. Encouraging students to justify their solutions as a part of the problem-solving activity provides teachers with a window into their students' thinking. Also, making students' reasoning visible to the classroom community may catalyze students to expand their argumentation and make use of forms of reasoning that habituates them better to articulate mathematical arguments (Maher & Martino, 1996; Maher, 2005; Maher et al., 2010). Importantly, teachers' attending to students' mathematical reasoning and facilitating discussions about the reasoning are central practices that produce