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The alignment of student fraction learning with textbooks in Korea and the United States

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ABSTRACT

A variety of international comparison studies of textbooks have been conducted on the assumption that a mathematics textbook is a critical factor in preparing students for mathematical achievement. The idea suggests that mathematics textbooks be written in a way that promotes students' cognitive developmental continuum of particular mathematical concepts. In the present study, we investigate how mathematics textbooks in Korea and the United States (in particular, Everyday Mathematics) entail students' learning progressions in terms of three operations (recursive partitioning, common partitioning, and distributive partitioning) that we assert undergird students' constructions of higher levels of fraction knowledge. Using the three operations and hierarchical relationships among them as an analytical framework, we select relevant topics with the operations and show how those topics are well aligned with our framework. We also discuss how the tasks included in the topics afford and constrain students' development of coherent fraction learning in terms of the framework.

1. Introduction

Textbooks play a crucial role for improvement in mathematics education (Reys, Reys, & Chavez, 2004). They "characterize not only the content but also advocate what students are to be able to do with that content—what mathematical behaviors are to be encouraged" (Schmidt, 2012, p. 143). Such significance of the role of textbooks in mathematics learning led to a wide variety of comparative studies of textbooks, which entail the general physical appearance (size of the textbooks, number of lessons), content organization (topics covered, sequencing of topics), aspects of problem design (cognitive demands, problem solving, representation types), and the mathematical content itself (e.g., Flanders, 1994; Fuson, Stigler, & Bartsch, 1988; Ginsburg, Leinwand, Anstrom, & Pollock, 2005; Haggarty & Pepin, 2002; Jones, 2004; Reys, Reys, & Koyama, 1996; Schmidt, McKnight, & Raizen, 1997; Stevenson & Bartsch, 1992; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). The studies have been conducted on the assumption that the 'mathematics textbook' is a critical factor in explicating students' mathematical achievement.

However, we argue that for international comparison studies in curriculum to be valid measures for students' mathematical achievement, the comparison of textbooks should involve thorough investigation of how those textbooks focus on students' learning progressions of particular subjects. Even though textbooks do not directly influence students' learning (Stein, Remillard, & Smith, 2007; Stigler & Hiebert, 1999), what students learn from the implemented curriculum is determined by how students make use of the intended curriculum (Remillard & Bryans, 2004). It is in this sense that one must design curriculum with the learners in mind.

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There are studies (e.g., Charalambous, Delaney, Hsu, & Mesa, 2010; Li, Chen, & An, 2009; Moseley, Yukri, & Ishida, 2006; Watanabe, 2007) that have attributed the poor performance in mathematics of U.S. students in international comparative studies to their curriculum's deficient focus on students' learning. U.S. mathematics textbooks have been criticized by researchers for the number of pages allotted to the textbooks with a great deal of repetition and review (Alajmi, 2012; Flanders, 1987; Jones, 2004; Schmidt et al., 1997; Schmidt, Hsing, McKnight, 2005; Valverde et al., 2002). In this line of argument, Schmidt et al. (2005) criticized the lack of coherence of U.S. mathematics curriculum as they defined curriculum coherence[1] as "articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential and hierarchical nature of the disciplinary content from which the subject matter derives" (p. 528).

In the United States, the Common Core State Standards for Mathematics (2010) was announced and an overwhelming number of states have adopted it to align with their existing state curriculum, suggesting a major change in their mathematics textbooks. The Common Core State Standards for Mathematics (CCSSM) was developed in an effort to incorporate research-based learning progressions of students' ways of learning mathematics by sequencing topics aligned with students' cognitive development and mathematical coherence. Thus, it calls for studies examining whether and how the up-to-date U.S. elementary mathematics textbooks that claim to reflect the CCSSM are well aligned with students' learning progressions.

Since Korean students have consistently performed well in mathematics at international comparative studies, several studies explore how Korean mathematics textbooks structure learning opportunities for their students. Son and Senk (2010), in their analysis of U.S. and Korean mathematics textbooks of fraction multiplication and division, identified five characteristics of Korean elementary school mathematics textbooks: 1) more time allocated to key topics; 2) more opportunities to learn both concepts and procedures simultaneously, rather than sequentially; 3) more opportunities to learn additional computational strategies; 4) more opportunities to engage in the use of various models in problems as well as in lessons (p. 136). They argued that the five identified features of Korean elementary mathematics textbooks could be the reasons that Korean students performed better than U.S. students in international comparative studies. However, both Son (2012) and Hong and Choi (2014) brought forth results that contradict the Son and Senk (2010) study. Although the two studies examined different mathematical topics (fraction addition and subtraction by Son, and quadratic equations by Hong & Choi) from Son and Senk, they argued that U.S. textbooks provide more opportunities for students to engage in mathematical problems with a higher level of cognitive demands than Korean textbooks. These contradictory results necessitate a framework that could better account for similarities and differences in textbook comparison studies. Especially, we assert, there is a need for a new framework for textbook analysis involving psychological aspects of students' ways of learning mathematics.

Given that few studies examined in detail how the curriculum reflected students' ways of learning mathematics, the current study investigates how mathematics textbooks entail students' learning progressions in terms of their structures at a macro level as well as with posed tasks or examples at a micro level. We review and compare one of the best-selling standards-based U.S. textbooks, *Everyday Mathematics* (4th edition published in 2016) and the current Korean government-designated elementary mathematics textbook, *The Mathematics* regarding fractions. Our analysis is based on context-specific, scheme-theoretic constructs (three partitioning operations) that provide fine-grained reports concerning students' development of fractional knowledge. To this end, we first introduce and discuss our theoretical framework to compare textbooks attending to cognitive behaviors or mental operations in students' developmental paths on fractions. Then we analyze mathematics textbooks from the two countries using the framework used to characterize learning opportunities structured in those textbooks. This is an initial step towards developing a framework for analyzing and understanding opportunities to learn mathematics on the basis of established models of students' developing particular mathematical concepts through intensive research of students' learning of these concepts. Moreover, we believe that our effort complements the past research by examining the "potential cognitive demands of mathematical tasks in textbooks not solely in relation to content following the logic of discipline but in relations between tasks and students, following the logic of the learners" (Sztajn, Confrey, Wilson, & Edgington, 2012, p.150). Engagement in cognitively challenging mathematical tasks leads to the greatest learning gains for students. However, not all tasks are created equal; they provide different opportunities for students to learn mathematics. In the next section, we discuss what and how comparison studies of curriculum were conducted in the field of mathematics education, followed by our theoretical framework and results of analysis.

2. Comparison studies of curriculum in mathematics education

Given the awareness that curricular differences across countries influence students' opportunities to learn mathematics (McDonnell, 1995), various types of curriculum analyses have been made to account for the differences in students' performance of mathematics from international comparative studies such as the Trends in International Mathematics and Science Study (TIMSS) and Program for International Student Assessment (PISA). Some large-scale efforts explored opportunities to learn mathematics by exploring the *implemented curriculum*, as interpreted by teachers in individual classrooms through surveying the teachers of tested students (Travers, 1993). According to Charalambous et al. (2010), such approach to examining textbooks is called the *contextual analysis* for "attending to the ways in which textbooks are used in instructional activities by either instructors or students" (p. 120). Acknowledging the validity issue of past research on measuring students' opportunities to learn through teachers' self-reports (Burstein et al., 1995), small-scale studies (Haggarty & Pepin, 2002; Schmidt, 1996; Stigler & Hiebert, 1999) closely examined the

[1] However, as Confrey, Shah, McGowan, Gianopulos, & Jones (2017) states, their focus was more on following the logic of mathematics as a discipline than on the empirical research on student learning.