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Causal set generator and action computer[☆]

William J. Cunningham [a,*], Dmitri Krioukov [b]

[a] Department of Physics, Northeastern University, 360 Huntington Ave. Boston, MA 02115, United States
[b] Departments of Physics, Mathematics, and Electrical & Computer Engineering, Northeastern University, 360 Huntington Ave. Boston, MA 02115, United States

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ABSTRACT

The causal set approach to quantum gravity has gained traction over the past three decades, but numerical experiments involving causal sets have been limited to relatively small scales. The software suite presented here provides a new framework for the generation and study of causal sets. Its efficiency surpasses previous implementations by several orders of magnitude. We highlight several important features of the code, including the compact data structures, the $O(N^2)$ causal set generation process, and several implementations of the $O(N^3)$ algorithm to compute the Benincasa–Dowker action of compact regions of spacetime. We show that by tailoring the data structures and algorithms to take advantage of low-level CPU and GPU architecture designs, we are able to increase the efficiency and reduce the amount of required memory significantly. The presented algorithms and their implementations rely on methods that use CUDA, OpenMP, x86 Assembly, SSE/AVX, Pthreads, and MPI. We also analyze the scaling of the algorithms' running times with respect to the problem size and available resources, with suggestions on how to modify the code for future hardware architectures.

Program summary
Program Title: Causal Set Generator and Action Computer
Program Files doi: http://dx.doi.org/10.17632/5k8wjrhgwh.1
Licensing Provisions: MIT
Programming Language: C++/CUDA, x86 Assembly
Nature of Problem: Generate causal sets and compute the Benincasa–Dowker action.
Solution Method: We generate causal sets sprinkled on a Lorentzian manifold by randomly sampling element coordinates using OpenMP and linking elements using CUDA. Causal sets are stored in a minimal binary representation via the `FastBitset` class. We measure the action in parallel using OpenMP, SSE/AVX and x86 Assembly. When multiple computers are available, MPI and POSIX threads are also incorporated.
Additional Comments: The program runs most efficiently with an Intel processor supporting AVX2 and an NVIDIA GPU with compute capability greater than or equal to 3.0.

1. Introduction

There exists a multitude of viable approaches to quantum gravity, among which causal set theory is perhaps the most minimalistic in terms of baseline assumptions. It is based on the hypothesis that spacetime at the Planck scale is composed of discrete "spacetime atoms" related by causality [1]. These "atoms", hereafter called elements, possess a partial order which encodes all information about the causal structure of spacetime, while the number of these elements is proportional to the spacetime volume—"Order + Number = Geometry" [2]. One of the first successes of the theory was the prediction of the order of magnitude of the cosmological constant long before experimental evidence [3], while one of the most recent significant advances was the definition of a statistical partition function for the canonical causal set ensemble Ω [4] based on the Benincasa–Dowker action [5]. This work, which examined the space of 2D orders $\Omega_{2D} \subseteq \Omega$ defined in [6], provided a framework to study phase transitions and measure observables, with paths towards developing a dynamical theory of causal sets from which Einstein's equations could possibly emerge in the continuum limit. Yet the progress along this path is partly blocked on numerical limitations. Since the theory is non-local, the combination of action computation running times, $O(N^3)$,

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* Corresponding author.
E-mail addresses: w.cunningham@northeastern.edu (W.J. Cunningham), dima@northeastern.edu (D. Krioukov).

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and thermalization times, $O(N^2)$, of Monte Carlo methods used to sample causal sets from the ensemble, result in $O(N^5)$ overall running times, limiting numerical experimentation to causal set sizes N of just tens of elements.

Here we present new fast algorithms to generate causal sets sprinkled onto a Lorentzian manifold and to compute the Benincasa–Dowker action, with an emphasis on how these algorithms are optimized by leveraging the computer's architecture and instruction pipelines. After providing a short background on causal sets and the Benincasa–Dowker action in Sections 1.1 and 1.2, we describe several algorithm implementations to generate causal sets in Section 2. Section 3 presents a highly optimized data structure to represent causal sets that speeds up the computation of the action, Section 4, by orders of magnitude. Section 5 presents an analysis of algorithms' running times as functions of the causal set size and available computational resources. We conclude with a summary in Section 6.

1.1. Causal sets

Causal sets, or locally-finite partially ordered sets, are the central object in the causal set approach to quantum gravity [1,7,8]. These structures are modeled as directed acyclic graphs (DAGs) with N labeled elements $(n_1, n_2, \ldots, n_N)$ and directed pairwise relations (n_i, n_j). If obtained by sprinkling onto a Lorentzian manifold, they approximate the manifold in the continuum limit $N \to \infty$. Lorentzian manifolds are $(d+1)$-dimensional manifolds with d spatial dimensions and one temporal dimension whose metric tensors $g_{\mu\nu}, \mu, \nu = 0, 1, \ldots, d$, have one negative eigenvalue [9,10]. These DAGs are a particular type of random geometric graph [11]: elements are assigned coordinates in time and d-dimensional space via a Poisson point process with intensity ξ, and they are linked pairwise if they are causally related, i.e., timelike separated in the spacetime with respect to the underlying metric (Fig. 1). As a side note, sprinkling onto a given Lorentzian manifold is definitely not the only way to generate random causal sets. The general definition of a causal set can be found in [1], and random causal sets also can be obtained by sampling from the canonical ensemble Ω [4], or more generally, from the ensemble of random partial orders $P_{n,p}$ [12], i.e., they can in general be treated as unlabeled partial orders. Due to the non-locality implied by the causal structure, causal sets have an information content which scales at least as $O(N^2)$ compared to that in competing theories of discrete spacetime which scales as $O(N)$ [13–15]. As a result, by using the causal structure information contained in these DAG ensembles, one can recover the spacetime dimension [16,17], continuum geodesic distance [18], differential structure [19–22], Ricci curvature [5], and the Einstein–Hilbert action [13,23–25], among other properties.

1.2. The Benincasa–Dowker action

In many areas of physics, the action (S) plays the most fundamental role: using the least action principle [26,27], one can recover the dynamic laws of the theory as the Euler–Lagrange equations that represent the necessary condition for action extremization $\delta S = 0$. In general relativity, from the Einstein–Hilbert (EH) action,

$$S_{EH} = \frac{1}{2}\int R\left(x^\mu\right)\sqrt{-g}\,dx^\mu\,, \quad (1)$$

where R is the Ricci scalar curvature and g is the metric tensor determinant, Einstein's field equations can be explicitly derived and then solved given a particular set of constraints [28]. Therefore, if one hopes to develop a dynamical theory of quantum gravity, one would hope that either the discrete action in the quantum theory converges to (1) in the large-N limit, as we find with the

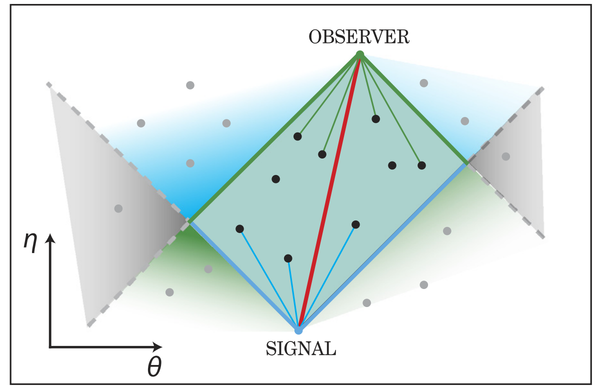

Fig. 1. The causal set as a random geometric graph. Elements of the causal set are sprinkled uniformly at random with intensity ξ into a particular region of spacetime, where η and θ respectively refer to the temporal and spatial coordinates in $(1+1)$ dimensions. Light cones, drawn by 45-degree lines in these conformal coordinates, bound the causal future and past of each element. When light cones of a pair of elements (shown in blue and green) overlap, the elements are said to be causally related, or timelike separated, as indicated by the bold red line. The black elements both to the future of the signal and to the past of the observer form the pair's Alexandroff set shown by the teal color. Not all pairwise relations are drawn. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Regge action for gravitation [29], or an interacting theory leads to an effective action, as we see with the Wilson action in quantum chromodynamics [30]. The numerical investigation of whether such a transition does indeed take place can be quite difficult: the quantum gravity scale is the Planck scale, so that if the convergence is slow, it may be extremely challenging to observe it numerically. This is indeed the case for the causal set discrete action, known as the Benincasa–Dowker (BD) action [5], which has been shown to converge slowly to the EH action in curved higher-dimensional spacetimes such as $(3+1)$-dimensional de Sitter spacetime [22,24].

The BD action was discovered in the study of the discrete d'Alembertian (B), i.e., the discrete covariant second-derivative approximating $\Box \equiv -\partial_t^2 + \nabla^2$, defined in $(1+1)$ dimensions, for instance, as

$$B\phi\left(x^\mu\right) = \frac{2}{l^2}\left(-\phi\left(x^\mu\right) + 2\left[\sum_{y\in L_1} - 2\sum_{y\in L_2} + \sum_{y\in L_3}\right]\phi\left(y^\mu\right)\right), \quad (2)$$

where $\phi(x^\mu)$ is a scalar field on the causal set, $l \equiv \xi^{-1/(d+1)}$ is the discreteness scale, and the ith order inclusive order interval (IOI) L_i corresponds to the set of elements $\{y\}$ which precede x with exactly $(i-1)$ elements $\{z_j\}$ within each open Alexandroff set, i.e., $y \prec \{z_j\} \prec x \, \forall y \in L_i$ and $|\{z_j\}| = i - 1$. In [5] it was shown that in the continuum limit, (2) converges in expectation to the continuum d'Alembertian plus another term proportional to the Ricci scalar curvature

$$\lim_{N\to\infty} \mathbb{E}\left[B\phi\left(x^\mu\right)\right] = \Box\phi\left(x^\mu\right) - \frac{1}{2}R\left(x^\mu\right)\phi\left(x^\mu\right)\,. \quad (3)$$

From (2) and (3) one can see when the field is constant everywhere, so that $\Box\phi(x^\mu) = 0$, then (2) converges to the Ricci curvature in the continuum limit, and therefore to the EH action when summed over the entire causal set. It was also shown in [5] that the expression for the BD action in $(1+1)$ dimensions is

$$S_{BD} = 2(N - 2n_1 + 4n_2 - 2n_3)\,, \quad (4)$$

where n_i is the abundance of the ith order IOI, i.e., the cardinality of the set L_i (Fig. 2). While (4) converges in expectation, any typical causal set tends to have a BD action far from the mean. This poses a