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A stochastic model for the evolution of bone metastasis: Persistence and recovery

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Abstract

Bone metastasis is a complex disease that modifies the natural dynamics of the bone remodeling process which is vital for maintaining a healthy bone structure. Nowadays, there are deterministic differential models that describe the bone metastasis evolution by considering interaction mechanisms between cancer cells and basic bone cells, such as osteoclasts and osteoblasts. Based on these deterministic models, a stochastic differential model is proposed to describe the bone metastasis evolution in order to analyze the effect of environmental fluctuations inclusions. Conditions for the existence and uniqueness of a nonnegative solution of the proposed stochastic model are given to characterize whether the invasion of cancer persists or is eradicated. Finally, we illustrate different dynamical behaviors of the stochastic solution by means of numerical simulations.

keywords: bone metastasis, osteoclasts, osteoblasts, stochastic differential equations, persistence, recovery.

1 Introduction

Over recent decades there has been considerable scientific interest in the bone remodeling process [4, 7, 26] considering that a malfunction of this physiological process would trigger serious problems in other vital systems. Osteoclasts and osteoblasts are a fundamental part of the basic multicellular unit (BMU) of bone remodeling. They perform the bone formation and resorption processes via a mutual paracrine-autocrine signaling [9, 12, 27]. The autocrine and paracrine singnaling is the form of BMU-cell communication by secreting different hormones (like osteoprotegerin and parathyroid) or chemical agents (like RANK, RANKL and TGF-β) in order to function as autoinducer. One of the reasons of miscommunication between the osteoclasts-osteoblasts is the appearance of cancer cells in the bone microenvironment [2, 20]. It occurs when these cells spread to the bone from some primary malignant tumor located in another part of the body; this process is known as bone metastasis. Cancer cells and the osteoclast and osteoblast cells maintain a complex biochemical interaction named the *vicious cycle* of bone metastasis and produce lethal bone diseases like the osteolytic (loss of bone mass) and osteoblastic (increment in bone mass) lesions [2, 18, 20, 23]. Thus, it is necessary to advance our understanding of the interactions between these three cell populations (osteoclast-osteoblast-cancer) in order to develop efficient medical treatments for bone metastasis.

Although the number of mathematical works for modeling the evolution of cancer cells in the bone microenvironment is considerably reduced, there are some mathematical models based on