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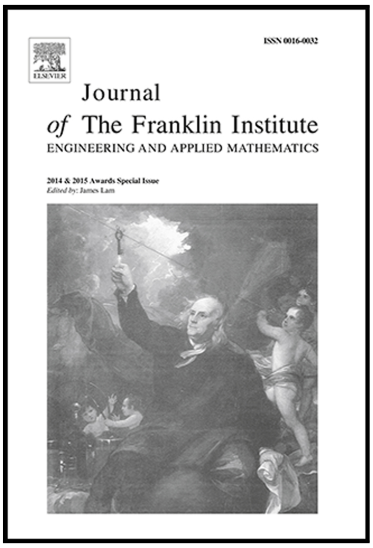

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On Frequency Distribution of Impulsive Feedback Control Times

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Abstract

The present work considers an impulsive feedback control problem governed by a system of ODEs defined on a class of isolated time scales. The control is activated once the unique solution's trajectory leaves a bounded connected domain. A periodicity of the controlled system's solution in the context of shift-invariant time scales are discussed. Making use of the Δ-measure theory, a cumulative distribution of the control times is introduced. Then with the help of dynamic inequalities on time scales such as Opial-type inequality, upper estimations for the occurrence frequency of the control times are obtained. A uniform distribution of the control times mod 1 is studied and a necessary condition for the uniformity of distribution of control times in terms of critical dynamic of the controlled system is obtained.

keywords Impulsive Feedback Control, Time Scales, Critical Dynamic, Δ-Measure, Frequency Distribution, Opial-type inequality, Uniform Distribution
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1 Introduction

Suppose a non-controlled dynamic is governed by an initial value problem

$$\begin{cases} \dot{x} = f(s, x(s)), & s \in \mathbb{R}^+ \\ x(0) = x_0 \in \Omega, \end{cases} \tag{1.1}$$

with $f : \mathbb{R}^+ \times \mathbb{R}^N \to \mathbb{R}^N$ satisfying existence and uniqueness conditions and $\Omega \subset \mathbb{R}^N$ is a connected bounded domain. The dimension N refers to the depth of our investigation which is reflected in the model. An impulsive (instantaneous) state-dependant control u is performed to keep the state within the domain Ω and it is defined as

$$u(s) = g(x(s))\delta\Big(H(x(s))\Big), \tag{1.2}$$

where $g : \mathbb{R}^N \to \mathbb{R}^N$ is continuous and $H(x) = 0$ defines an $(N-1)$-dimensional manifold $\partial\Omega$.