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Bayesian prediction for a jump diffusion process – With application to crack growth in fatigue experiments

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A R T I C L E I N F O

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A B S T R A C T

In many fields of technological developments, understanding and controlling material fatigue is an important point of interest. This article is concerned with statistical modeling of the damage process of prestressed concrete under low cyclic load. A crack width process is observed which exhibits jumps with increasing frequency. Firstly, these jumps are modeled using a Poisson process where two intensity functions are presented and compared. Secondly, based on the modeled jump process, a stochastic process for the crack width is considered through a stochastic differential equation (SDE). It turns out that this SDE has an explicit solution. For both modeling steps, a Bayesian estimation and prediction procedure is presented.

1. Introduction

In constructional engineering, material fatigue is a relevant topic of research because it is important to predict the lifetime of, for example, bridges. Experiments in this field, especially under low loadings, are very rare because they are very expensive and take very long, at times over several months. Maurer and Heeke [13] carried out five experiments where prestressed concrete beams with initial cracks have been put under cyclic load. Recently, two new experiments were conducted, see Heeke et al. [6]. All seven data series are considered here. Each prestressed concrete beam contains 35 tension wires which usually break at different time points. Therefore, the resulting crack widths which can be seen in Fig. 1 for two of the experiments, exhibit jumps with increasing frequency that influence the crack growth process substantially. Structure-borne noise measurements during the experiments provide information concerning the break times of the tension wires which match the observed jumps in the crack width data. This finding has important implications for the estimation procedure.

A model often used for crack growth in the engineering literature is the deterministic process defined by the Paris–Erdogan equation

$$\frac{da}{dN} = C(\Delta K)^m \tag{1}$$

where a denotes the crack length, N the number of cycles corresponding to the crack length a, while C and m are material constants. ΔK denotes the stress intensity factor range that depends on the square root of a. That leads to an autoregressive process whose differences increase with the growth of the process itself. For further information see, for example, Sobczyk and Spencer [22]. Because any process resulting from this equation will be deterministic and, therefore, will not describe the uncertainties of the growth process, this approach will be extended in the remainder.

A diffusion model would be one extension of the ordinary differential equation in (1). A Bayesian approach for growth diffusions is investigated in Hermann et al. [8] within a hierarchical model and applied to a data set of experiments in aluminum alloy which is a homogeneous material in contrast to concrete. Therefore, we need some additional approach for modeling the irregular jumps in the concrete data. In Heeke et al. [6], the underlying jump process is modeled by a nonhomogeneous Poisson process (NHPP), which is embedded in a nonlinear regression model. Of course, other jump processes are possible, but the NHPP is a good starting point because we observe very few jumps and cannot, therefore, estimate many parameters. Since we observe an increasing frequency of the jumps, a homogeneous process would not be able to predict reliably. Therefore, a NHPP is reasonable. A two-parameter approach for the intensity was used in Heeke et al. [6] and provided good results. Instead of the nonlinear regression model for the concrete crack width, in the following we introduce a more intuitive process model; it is defined by a stochastic differential equation (SDE) that extends the Paris–Erdogan equation (1) and includes the NHPP which fits the jumps in the data. In the

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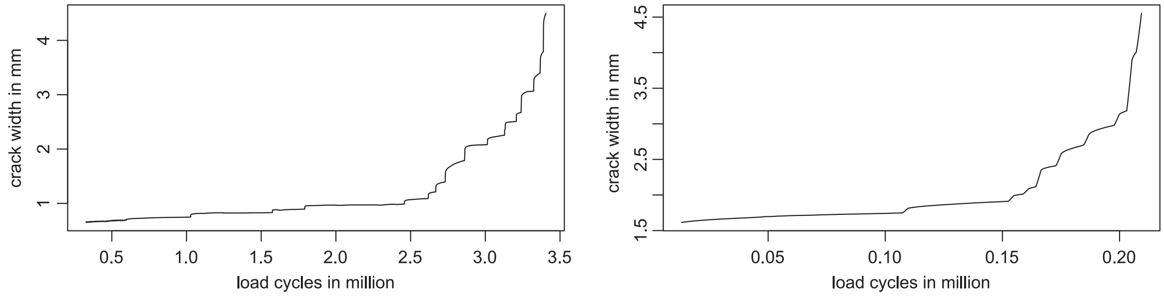

Fig. 1. Crack width data resulting from the first two experiments of Maurer and Heeke [13].

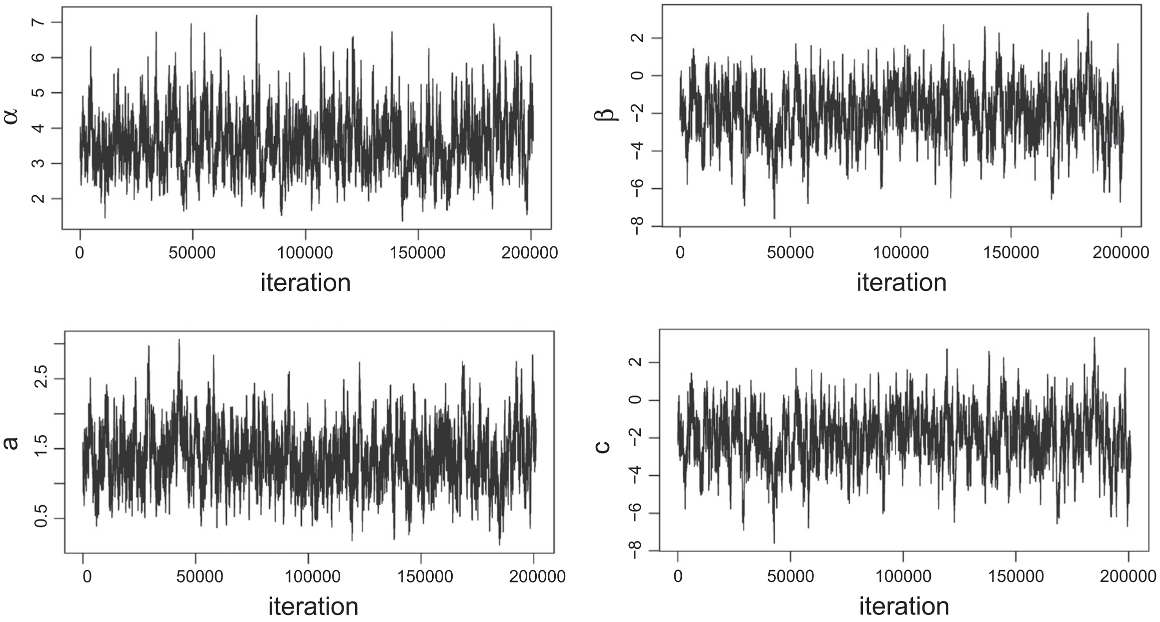

Fig. 2. Markov chains of the MH algorithm to simulate from the posterior distributions for α and β in the case of $\Lambda_\xi(t) = \left(\frac{t}{\beta}\right)^\alpha$ (top) and for a and c in the case of $\Lambda_\xi(t) = \exp(at + c) - \exp(c)$ (bottom) of the NHPP.

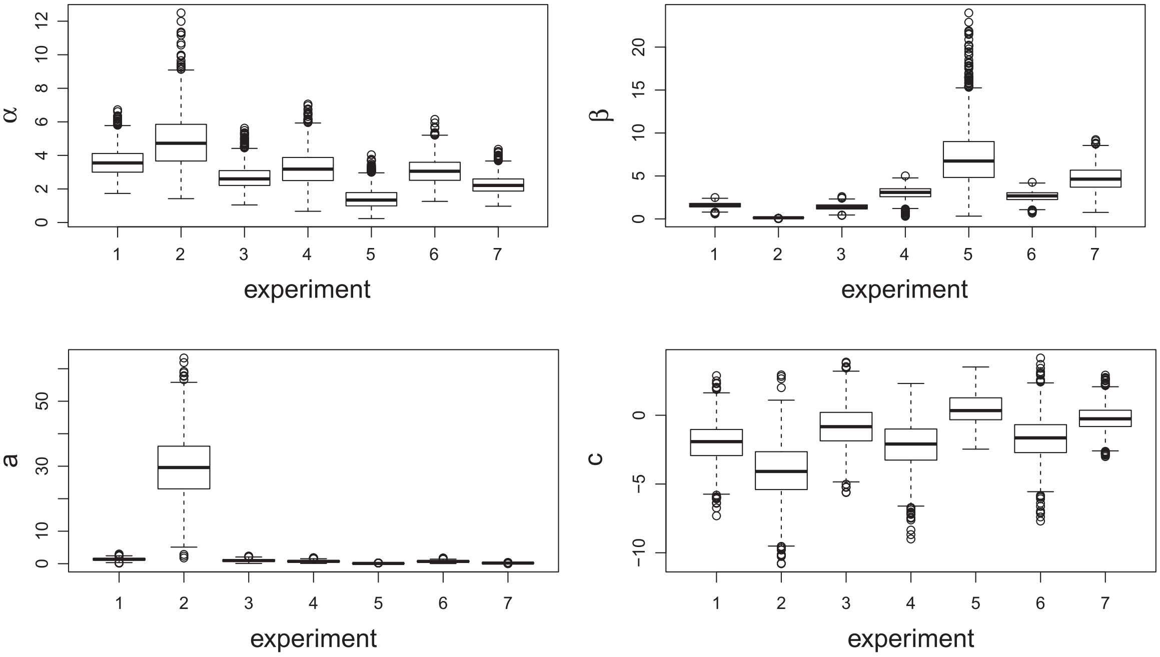

Fig. 3. Boxplots of the posterior distributions for α and β in the case of $\Lambda_\xi(t) = \left(\frac{t}{\beta}\right)^\alpha$ (top) and for a and c in the case of $\Lambda_\xi(t) = \exp(at + c) - \exp(c)$ (bottom) of the NHPP.

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