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Inflationary cosmology and the scale-invariant spectrum

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ABSTRACT

The claim of inflationary cosmology to explain certain observable facts, which the Friedmann-Roberston-Walker models of 'Big-Bang' cosmology were forced to assume, has already been the subject of significant philosophical analysis. However, the principal empirical claim of inflationary cosmology, that it can predict the scale-invariant power spectrum of density perturbations, as detected in measurements of the cosmic microwave background radiation, has hitherto been taken at face value by philosophers.

The purpose of this paper is to expound the theory of density perturbations used by inflationary cosmology, to assess whether inflation really does predict a scale-invariant spectrum, and to identify the assumptions necessary for such a derivation.

The first section of the paper explains what a scale-invariant power-spectrum is, and the requirements placed on a cosmological theory of such density perturbations. The second section explains and analyses the concept of the Hubble horizon, and its behaviour within an inflationary space-time. The third section expounds the inflationary derivation of scale-invariance, and scrutinises the assumptions within that derivation. The fourth section analyses the explanatory role of 'horizon-crossing' within the inflationary scenario.

1. Introduction

In the past couple of decades, inflationary cosmology has been the subject of trenchant criticism in some quarters. The criticism has come both from physicists (Penrose, 2004, 2010, 2016; Steinhardt, 2011), and philosophers of physics (Earman, 1995, Chapter 5; Earman & Mosterin, 1999). The primary contention is that inflation has failed to deliver on its initial promise of supplying cosmological explanations which are free from dependence on very special initial conditions.

Inflation was initially promoted as a theory which explained certain observable astronomical facts that the Friedmann-Roberston-Walker (FRW) models of 'Big-Bang' cosmology were forced to assume, or explain by means of fine-tuned initial conditions (Guth, 1981). The most prominent examples of this were dubbed the 'horizon problem' and the 'flatness problem', (McCoy, 2015).

In the first case, points in the cosmic microwave background sky with a large angular separation, possess very similar temperatures despite the fact that there was insufficient time in an FRW model for these regions to have causally interacted before the time of 'recombination', when the photons in the background radiation effectively decoupled from the matter. In the second case, it was pointed out that the current value of the density parameter Ω_0 is very close to 1, despite the fact that $\Omega_0 = 1$ is an unstable fixed point of the FRW dynamics (Smeenk, 2012).

At first sight, inflation was able to explain these facts as the result of evolutionary processes rather than initial conditions. Its failure to deliver on this promise is rooted in the fact that inflation was also tasked with reproducing the spectrum of density perturbations ultimately responsible for seeding galaxy formation, ('structure formation'). In order to produce the correct statistics, the scalar field responsible for the hypothetical period of exponential expansion had to be parameterised in a fashion inconsistent with any candidate field available in a Grand Unified Theory of particle physics, (Smeenk, 2012).

It became clear that the predictions of inflation were extremely sensitive to the type of scalar field chosen, and to the initial conditions of that field: "The original models of inflationary cosmology…predicted an amplitude of density fluctuations that was too high by several orders of magnitude. To get the right order of magnitude, the false vacuum plateau of the inflaton field has to be very flat … For a slow roll potential, the ratio of the change in potential to the change in the scalar field must be less than 10^{-6} – 10^{-8}, and for the potential used in the 'extended' inflationary

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scenario the ratio must be less than 10^{-15}," (Earman & Mosterin, 1999).

Moreover, once inflation was no longer tied down to the world of particle physics, a cornucopia of different models was unleashed: "Martin, Ringeval and Vennin (2014a) have catalogued and analyzed a total of 74(!) distinct inflaton potentials that have been proposed in the literature: all of them corresponding to a minimally coupled, slowly-rolling, single scalar field driving the inflationary expansion. And a more detailed Bayesian study (Martin, Ringeval, Trotta, & Vennin, 2014b), expressly comparing such models with the Planck satellite's 2013 data about the CMB, shows that of a total of 193(!) possible models — where a 'model' now includes not just an inflaton potential but also a choice of a prior over parameters defining the potential — about 26% of the models (corresponding to 15 different underlying potentials) are favored by the Planck data. A more restrictive analysis (appealing to complexity measures in the comparison of different models) reduces the total number of favoured models to about 9% of the models, corresponding to 9 different underlying potentials (though all of the 'plateau' variety)," (Azhar & Butterfield, 2017).

Whilst inflationary cosmologists have retreated somewhat from the claim that their theory is independent of initial conditions, faith in the theory has instead been built on its empirical success. The theory, it is claimed, predicts that the spectrum of density perturbations is (almost) scale-invariant, and observations of the cosmic microwave background radiation verify this prediction.

The critics of inflation are able to point out that the class of inflationary models is so general that it could explain just about any empirical data. But the claim that inflation predicts a scale-invariant spectrum is generally accepted without reservation or further examination.

The purpose of this paper is to expound the theory of density perturbations used by inflationary cosmology, and to assess whether inflation really does predict a scale-invariant spectrum.

The first section of the paper explains what a scale-invariant power-spectrum is, and the requirements placed on a cosmological theory of such density perturbations. The second section explains and analyses the concept of the Hubble horizon, and its behaviour within an inflationary space-time. The third section expounds the inflationary derivation of scale-invariance, and scrutinises the assumptions within that derivation. The fourth section analyses the explanatory role of 'horizon-crossing' within the inflationary scenario.

2. Perturbations and the power spectrum

Given a scalar field $\rho(x)$ representing the density of matter, with a mean density $\langle\rho\rangle$, the *fluctuation field* (or 'perturbation field') $\delta_\rho(x)$ is defined by

$$\delta_\rho(x) = \rho(x) - \langle\rho\rangle \,, \tag{2.1}$$

and the *contrast field* is defined by

$$\frac{\delta_\rho(x)}{\langle\rho\rangle} = \frac{\rho(x) - \langle\rho\rangle}{\langle\rho\rangle} \,. \tag{2.2}$$

Let us assume, for convenience, that we are working with a foliation of space-time in which the global topology of space is $\mathbb{R}^3$, and the spatial curvature is zero. The fluctuation field can be expressed as an inverse Fourier transform:

$$\delta_\rho(\mathbf{x}) = \frac{1}{(2\pi)^3}\int A(\mathbf{k})e^{i\mathbf{x}\cdot\mathbf{k}}d\mathbf{k} \,, \tag{2.3}$$

where $(\mathbf{x},\mathbf{k})\mapsto\mathbf{x}\cdot\mathbf{k}$ is the Euclidean inner product.

This expresses the fluctuation field as the superposition of a spectrum of wave-like 'modes'. The $\mathbf{k}$-th mode is $e^{i\mathbf{x}\cdot\mathbf{k}}$, and the amplitude of the $\mathbf{k}$-th mode is $A(\mathbf{k})$. The wavelength λ of the $\mathbf{k}$-th mode is related to the wave-vector $\mathbf{k}$ by:

$$\lambda = \frac{2\pi}{|\mathbf{k}|} \,, \tag{2.4}$$

where $|\mathbf{k}|$ is the wave-number. Hence, long wavelength perturbations correspond to small wave-numbers, and short wavelength perturbations correspond to large wave-numbers.

Now, while the mean value of δ_ρ is zero, $\langle\delta_\rho\rangle = 0$, the mean of its square-value is non-zero, $\langle\delta_\rho^2\rangle \neq 0$. The mean of the square-value is simply the variance $\sigma^2_{\delta_\rho}$ in the fluctuation field:

$$\sigma^2_{\delta_\rho} = \left\langle \delta_\rho^2 \right\rangle \neq 0 \,. \tag{2.5}$$

The variance can be expressed in terms of the amplitudes of the perturbational modes as follows:

$$\sigma^2_{\delta_\rho} = \frac{1}{(2\pi)^3}\int |A(\mathbf{k})|^2 d\mathbf{k} \tag{2.6}$$

Assuming the perturbations to the density field are sampled from a homogeneous and isotropic random field, then the random field will be spherically symmetric about any point, and this 3-dimensional integral over the space of wave-vectors $\mathbf{k}$ can be simplified into an integral over wave-numbers $k = |\mathbf{k}|$:

$$\sigma^2_{\delta_\rho} = \frac{1}{(2\pi)^3}\int_0^\infty |A(k)|^2 4\pi k^2 \, dk \,, \tag{2.7}$$

where $4\pi k^2$ is the surface area of a sphere of radius k in the space of wave-vectors.

The square of the modulus of the amplitudes is called the *power spectrum*:

$$P_\rho(k) = |A(k)|^2 \,. \tag{2.8}$$

Hence, the variance can be expressed as

$$\sigma^2_{\delta_\rho} = \frac{1}{2\pi^2}\int P_\rho(k)k^2 dk \,. \tag{2.9}$$

The significance, then, of the power spectrum, is that it determines the contribution of the mode-k perturbations to the total variance.

According to inflationary cosmology, the power spectrum of the density perturbations is given by a power law:

$$P_\rho(k) = Ak^n \tag{2.10}$$

where A is some constant (not to be confused with the Fourier coefficients above), and the exponent n is called the *spectral index*. Inflationary cosmology purportedly predicts that $n \approx 1$. A power spectrum with such an exponent is said to be (approximately) *scale-invariant*.[1]

Note that $P_\rho(k) \sim k$ entails that the amplitude of the perturbations increase with k. Greater wave-numbers correspond to shorter

[1] The exponent can only be constant over a finite range of wave-numbers. The convergence of (2.9) requires that $n > -3$ for $k \to 0$, and $n < -3$ for $k \to \infty$, (Coles & Lucchin, 2002, p. 265). Alternatively, the power-law form itself might only be valid over a finite range of wave-numbers.

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