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Time isotropy, Lorentz transformation and inertial frames

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A B S T R A C T

Homogeneity of Euclidean space and time, spatial isotropy, principle of relativity and the existence of a finite speed limit (or its variants) are commonly believed to be the only axioms required for developing the special theory of relativity (Lorentz transformations). In this paper, however, it is pointed out that the Lorentz transformation for a boost cannot actually be derived without the explicit assumption of time isotropy (viz. time-reversal symmetry) which is logically independent of the other postulates of relativity. Postulating time isotropy also restores the symmetry between space and time in the postulates of relativity (i.e. time and space share the same symmetries then). Time isotropy also helps explain naturally one key general feature of the fundamental physical laws, viz. their time-reversal symmetry. But inertial frames are defined in influential texts as frames having space-time homogeneity and spatial isotropy only. Inclusion of time isotropy in that definition is thus suggested.

1. Introduction

Since its advent, the foundations of special theory of relativity have often been reviewed in order to find the minimum number of axioms required to develop the theory rigorously [1]. These minimalist attempts claim to have founded special theory of relativity on the assumptions of *homogeneity of space and time*, *spatial isotropy* and *principle of relativity*, thus making Einstein's 2nd postulate (Einstein, 1905), that of the constancy of speed of light in free space, redundant. Actually, these assumptions give two possible kinematics viz. the Galilean transformation and the Lorentz transformation (Drory, 2015). From Lorentz transformation we know that a space coordinate and time mix symmetrically. With hindsight, therefore, it seems intriguing that time isotropy should be absent as a postulate of special theory of relativity when spatial isotropy is explicitly postulated and homogeneity is assumed in both space and time. By time isotropy we mean the equivalence of the two time directions viz. past and future[2]. Time isotropy is the symmetry by which the fundamental physical laws are time-reversal invariant.

Exclusion of time isotropy becomes all the more puzzling when spatial isotropy and space-time homogeneity are taken to be the only defining properties of inertial frames (Landau and Lifshitz, 1976, p. 5). Inertial frames are devised to make the description of physics simplest. Space and time in these frames, therefore, should possess the highest possible symmetry. Since time is independent of space by definition, time isotropy is independent of spatial isotropy and space-time homogeneity. Hence, time isotropy should also be a defining attribute of inertial frames along with spatial isotropy and space-time homogeneity. Moreover, the fundamental classical laws are time-reversal symmetric. This becomes natural if time isotropy is considered a defining property of inertial frames (just as the rotational invariance of physical laws is considered a natural consequence of spatial isotropy).

In the following we derive Lorentz transformation and Galilean transformation from principle of relativity, space-time homogeneity, spatial isotropy and time isotropy. The main purpose of this article is to point out that principle of relativity, space-time homogeneity, spatial isotropy and the existence of a finite speed limit indeed do not exhaust all the postulates of special theory of relativity, since Lorentz transformation for a boost cannot be logically derived from them alone without appealing to time isotropy. How then did the authors of the abovementioned literature get to Lorentz transformation (or Galilean transformation for that matter) without assuming time isotropy? The logical fallacies that allowed

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[1] See Ref. Berzi and Gorini, 1969, Feigenbaum, 2008, Pal, 2003 and the references listed therein.

[2] Time isotropy or time-reversal symmetry does not violate causality. Time reversal merely changes the terms, what was “cause” becomes “effect” and what was “effect” becomes “cause”. Their one-to-one relation (causal connection) remains intact and their time-order remains frame-independent. Special theory of relativity does not possess an inherent arrow of time.(Leggett, 1987)

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them to do so will contextually be remarked upon in due course by comparing with our proposed development.

2. Deriving Lorentz transformation

2.1. Definitions and axioms

1. By "frame" in the following, we mean inertial frames with their own Cartesian triads and time. It is supposed that space is Euclidean in these frames.
2. Inertial frames are defined as frames having space-time homogeneity, spatial isotropy and time isotropy.
3. Principle of relativity is postulated to hold between inertial frames.

2.2. 1st step: Linearity of transformations from space-time homogeneity

Physics deals with laws of nature which deal with change of state in turn. Description of change of state in any frame is in terms of space and time. Consider now any two such frames $S\,(x,y,z,t)$ and $S'(x',y',z',t')$ linked by single-valued transformation functions,

$$x' = x'(x,y,z,t), \tag{1}$$

$$y' = y'(x,y,z,t), \tag{2}$$

$$z' = z'(x,y,z,t) \text{ and} \tag{3}$$

$$t' = t'(x,y,z,t), \tag{4}$$

and their inverses. The three rectilinear space-coordinates and time are represented by their usual symbols. These functions must be differentiable everywhere in the space-time continuum (i.e. the domain of the transformation functions), as otherwise, the existence of singular point(s) would violate space-time homogeneity. Differentiating Eqs. (1)–(4) we get

$$\Delta x' = \frac{\partial x'}{\partial x}\Delta x + \frac{\partial x'}{\partial y}\Delta y + \frac{\partial x'}{\partial z}\Delta z + \frac{\partial x'}{\partial t}\Delta t, \tag{5}$$

$$\Delta y' = \frac{\partial y'}{\partial x}\Delta x + \frac{\partial y'}{\partial y}\Delta y + \frac{\partial y'}{\partial z}\Delta z + \frac{\partial y'}{\partial t}\Delta t, \tag{6}$$

$$\Delta z' = \frac{\partial z'}{\partial x}\Delta x + \frac{\partial z'}{\partial y}\Delta y + \frac{\partial z'}{\partial z}\Delta z + \frac{\partial z'}{\partial t}\Delta t \text{ and} \tag{7}$$

$$\Delta t' = \frac{\partial t'}{\partial x}\Delta x + \frac{\partial t'}{\partial y}\Delta y + \frac{\partial t'}{\partial z}\Delta z + \frac{\partial t'}{\partial t}\Delta t. \tag{8}$$

According to space-time homogeneity, no point in space or time is preferred. So no choice of origin is physically favoured and space and time intervals are the only concepts that have any physical relevance. "Equality of space-time intervals" must therefore be frame-independent and objective. Equal space-time intervals in S thus should correspond to equal space-time intervals in S'. It is important to note that a space-time interval in any frame (e.g. S) is fully represented by the whole matrix $(\Delta x\ \ \Delta y\ \ \Delta z\ \ \Delta t)$ (and not by any particular function of its elements). For Euclidean space, the equality of two space-time intervals implies the equality of the corresponding matrices[3]. All these mean that Eqs. (5)–(8) must be independent of (x,y,z,t) which is possible only if the transformation functions in Eqs. (1)–(4) are linear in (x,y,z,t). Rewriting Eqs. (1)–(4) in matrix form thus, we get

$$\begin{pmatrix} x' \\ y' \\ z' \\ t' \end{pmatrix} = \boldsymbol{T}\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} + \begin{pmatrix} o_x \\ o_y \\ o_z \\ o_t \end{pmatrix}, \tag{9}$$

where $\boldsymbol{T}$ is a transformation matrix independent of (x,y,z,t), and the rightmost column matrix is a constant dependent on the choice of origin. For points fixed in S' space, $\frac{dx'}{dt}=0$, $\frac{dy'}{dt}=0$, $\frac{dz'}{dt}=0$ as seen from S. Hence, differentiating the first three rows of Eq. (9) with respect to t,

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \boldsymbol{\Gamma}\begin{pmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{pmatrix} + \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \tag{10}$$

where $\boldsymbol{\Gamma}$ is some matrix and the rightmost column is constant. Solving Eq. (10) for $\begin{pmatrix} \frac{dx}{dt} & \frac{dy}{dt} & \frac{dz}{dt} \end{pmatrix}^T$ we find that S' moves with a constant velocity with respect to S. Therefore, the inertial frames move with uniform velocity relative to each other.

2.3. 2nd step: Form of a Lorentz boost

Any transformation taking one inertial frame to another is called a Lorentz transformation. *A boost is that particular Lorentz transformation which exists by virtue of relative velocity only.* In this section, we try to find the general form of the boost (transforming inertial frame S to S') that comes out from the sole stipulation of a given relative velocity vector.

Let the velocity of S' relative to S be $\boldsymbol{v}$. From Eq. (9),

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \boldsymbol{\Gamma}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + t\begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}. \tag{11}$$

$\boldsymbol{\Gamma}$ was introduced in Eq. (10). Since the left hand side of Eq. (11) is a vector (position vector in S') so must be the right hand side. $\boldsymbol{\Gamma}$ therefore acts as a linear operator (this idea is inspired from Ref. (Feigenbaum, 2008)). The last column vector in Eq. (11) is independent of (x,y,z,t). It, therefore, must be along the velocity vector (i.e. $\boldsymbol{v}$) of S'relative to S, since all the other directions are equivalent by spatial isotropy in S. The next problem is to find how the time of S transforms into that of S'. From Eq. (9) again

$$t' = et + \boldsymbol{V}\cdot\boldsymbol{r} \tag{12}$$

where e is a scalar and $\boldsymbol{V}$ is some vector independent of $\boldsymbol{r}$. By spatial isotropy again, $\boldsymbol{V}$ must be along $\boldsymbol{v}$, i.e.

$$\boldsymbol{V} = f\boldsymbol{v} \tag{13}$$

for some scalar f independent of (x,y,z,t).

[3] For non-Euclidean homogeneous space, equal space intervals (infinitesimal position vectors) are realised by parallel transport during which the spatial curvature may induce a change in the spatial components $(\Delta x, \Delta y, \Delta z)$. For these spaces therefore, the affine nature of space-time transformations (Eq. (9)) cannot be inferred from the frame-independence of "equality of space-time intervals" and Eq. (5)–(8). See section 3.1 "Space-Time 'Homogeneity'" in Mamone-Capria, 2016.

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