New fractional nonlinear integrable Hamiltonian systems

Oksana Ye. Hentosh, Bohdan Yu. Kyshakevych, Denis Blackmore, Anatolij K. Prykarpatski

PII: S0893-9659(18)30288-X
DOI: https://doi.org/10.1016/j.aml.2018.08.009
Reference: AML 5620

To appear in: *Applied Mathematics Letters*

Received date : 16 June 2018
Revised date : 11 August 2018
Accepted date : 12 August 2018

Please cite this article as: O.Y. Hentosh, B.Y. Kyshakevych, D. Blackmore, A.K. Prykarpatski, New fractional nonlinear integrable Hamiltonian systems, Appl. Math. Lett. (2018), https://doi.org/10.1016/j.aml.2018.08.009

NEW FRACTIONAL NONLINEAR INTEGRABLE HAMILTONIAN SYSTEMS

OKSANA YE. HENTOSH, BOHDAN YU. KYSHAKEVYCH, DENIS BLACKMORE,
AND ANATOLIJ K. PRYKARPATSKI

Abstract. We have constructed a new fractional pseudo-differential metrized operator Lie algebra on the axis, enabling within the general Adler–Kostant–Symes approach the generation of infinite hierarchies of integrable nonlinear differential-fractional Hamiltonian systems of Korteweg-de Vries, Schrödinger and Kadomtsev–Petviashvili types. Using the natural quasi-classical approximation of the metrized fractional pseudo-differential operator Lie algebra, we construct a new metrized fractional symbolic Lie algebra and related infinite hierarchies of integrable mutually commuting fractional symbolic Hamiltonian flows, modeling Benney type hydrodynamical systems.

1. Introduction

Nonlinear dynamical systems described by differential-fractional equations (fractional in space, continuous in time) are currently of great interest [9, 19, 20, 23]. Deriving suitable analogs of integrable nonlinear differential-fractional equations for known integrable dynamical systems [10, 6, 5, 21, 37] in partial derivatives is especially important. A derivation scheme, based on certain fractional-operator linear spectral problems that generate integrable nonlinear differential-fractional equations, was suggested in [7, 13, 26]. The related fractional differentiation was defined via the resolvent operator in Seeley [32], which was also devised in [35] for an algebra of pseudo-differential operator symbols. The corresponding construction [26], based on the Adler–Kostant–Symes [2, 34, 31, 6, 5] scheme, enabled the construction of new integrable hierarchies of Hamiltonian differential-difference and fractional dynamical systems.

We show that the Lie-algebraic Adler–Kostant–Symes scheme can be also applied to the fractional pseudo-differential metrized operator algebra based on the Riemann–Liouville [9] fractional derivative, giving rise to infinite hierarchies of new integrable differential-fractional Hamiltonian systems of the Korteweg–de Vries (KdV) and nonlinear Schrödinger (NLS) types. The natural quasi-classical approximation [4, 8, 41] of the basic metrized fractional pseudo-differential operator Lie algebra is used to devise a new metrized fractional symbolic Lie algebra and construct related infinite hierarchies of integrable mutually commuting fractional symbolic KdV and NLS type Hamiltonian flows, modeling the well-known [14, 16, 17, 18, 29, 41] Benney type hydrodynamical systems.

2. Fractional analysis setting

We begin with an associative functional algebra $(A;+,\cdot)$, $A = W_2^\infty(\mathbb{R};\mathbb{C})\cap W_\infty^\infty(\mathbb{R};\mathbb{C})$, over $\mathbb{C}$, endowed with the standard Riemann–Liouville fractional derivative $D^\alpha : A \to A$ satisfying the semigroup property $D^\alpha(D^\beta) = D^{\alpha+\beta}$ [9]. Define on A the following symmetric bilinear form

$$(a,b) := \int_{\mathbb{R}} a(x)b(x)dx, \tag{2.1}$$

where $a, b \in A$. The adjoint with respect to (2.1) of the derivative map satisfies

$$(D_*^\alpha a, b) = (a, D^\alpha b) \tag{2.2}$$

for any $a, b \in A$ and acts as

$$D_*^\alpha a(x) := \frac{(-1)^n}{\Gamma(n-\alpha)} \int_{-\infty}^{x} \frac{da^{(n-1)}(y)}{(y-x)^{\alpha-n+1}} \tag{2.3}$$

for any $f \in A$ and $n = [\operatorname{Re}\alpha] + 1 \in \mathbb{Z}_+$.

1991 *Mathematics Subject Classification.* 35A30, 35G25, 35N10, 37K35, 58J70,58J72, 34A34;

Key words and phrases. fractional pseudo-differential metrized operator Lie algebra, fractional symbolic metrized functional Lie algebra, ad-invariant trace-functional, R-structure, Adler–Kostant–Symes approach, Lie–Poisson structure, Casimir invariants, fractional Korteweg–de Vries type equations, fractional Nonlinear Schrödinger type equations .