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Three-dimensional vibration analysis of curved and twisted beams with irregular shapes of cross-sections by sub-parametric quadrature element method

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ABSTRACT

This paper presents a novel three-dimensional (3D) sub-parametric quadrature element (SP-QE) method for solving the coupled dynamic behavior of curved and pre-twisted beamlike structures with irregular shapes of cross-section. The technique is an extension of the existing quadrature element method (QEM) with regular shapes by mapping the irregular solid into a regular cube. Detailed formulations are worked out. Beams with rectangular, circular, elliptical and airfoil cross-sections, various curvature and pre-twist rates, and different boundary conditions are investigated. Either Serendipity elements or Lagrange elements are considered in the mapped regular domain. Convergence studies are carried out to show the computational performance of the proposed elements. Results are compared either with the existing 3D spectral-Tchebychev (3D-ST) solutions or with the finite element data. It is shown that the proposed method can yield accurate solutions with small number of degrees of freedom. Consistent or lumped mass matrix affects little on the accuracy of solutions. Therefore, the element with lumped mass matrix can be efficiently used in dynamic analysis of solids with regular and irregular shapes.

1. Introduction

Curved or/and pre-twisted beamlike structures with regular and irregular shapes of cross-sections find many applications in aeronautical engineering and architectural areas. Understanding and predicting their dynamic behavior accurately is highly crucial in design the structures. A vast body of the literature exists and is well documented in a review paper written by Zhao et al. [1]. Over two hundreds papers have been reviewed.

The out-of-plane free vibration of horizontally curved beams with variable curvature is analyzed by Lee et al. [2]. Their theoretical frequencies are verified by experimental data. Rajasekaran [3] uses a new differential transformation method to study the static and out of plane vibration of axially functionally graded tapered curved beams. A differential–integral quadrature method (DIQM) is used by Mohamed et al. [4] to predict nonlinear free and steady state forced vibrations of curved beam with both ends clamped. Euler–Bernoulli kinematics assumptions including mid-plane stretching are employed. Results agree well with analytical solutions.

Since the dynamics of curved or/and pre-twisted beamlike structures present coupled three dimensional (3D) motions, such as the coupling of bending-axial (in-plane) motion or/and bending–twisting (out-of-plane) motion, simple beam

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theories may not describe their dynamic behavior accurately. Thus, a 3D analysis method is necessary to accurately predict the dynamic behavior of curved or/and pre-twisted beams with regular and irregular shapes of cross-section [5–7].

While conventional finite-element method (FEM) is very powerful for structural analysis, some essential shortcomings exist, such as the necessity of using a large number of solid elements to model the structures. In general FEM is computationally expensive and time consuming to perform a 3D analysis. Therefore, some computationally more efficient continuous or numerical methods are required, especially, in the design stage when optimum geometries of curved or/and pre-twisted beamlike structures are to be found.

Bediz and Aksoy [5] use 3D spectral-Tchebychev (3D-ST) technique to predict the free vibration of curved and pre-twisted beams with rectangular and airfoil shapes of cross-sections. The solution function is expressed in terms of scaled Tchebychev polynomials and then defined by giving its values at the Gauss–Lobatto points. The 3D-ST method, having the merit of symmetric stiffness and mass matrices, may be regarded as a weighted residual method or Galerkin method. Accurate results can be obtained by the 3D-ST method with a small number of terms in the assumed displacement functions.

Analytical solutions for vertical, torsional, radial, and axial responses of the curved beam subjected to three-directional moving loads are reported by Li and Ren [6]. To get analytical solutions, they use Galerkin method to discretize the partial differential equations and modal superposition method to decouple the ordinary differential equations. Zhang et al. [7] used the isogeometric approach to analyze the 3D behavior of the curved beams. The effectiveness of the isogeometric approach is demonstrated on a variety of curved structural geometries of varying complexities. To handle the complex geometrical forms, new structural analysis methods are needed to accurately and efficiently analyze beamlike structures with regular and irregular shapes [7].

"Along with the ever-growing advancement of faster computing machines, the research into the development of new methods for numerical solution of problems in engineering and physical sciences also is an ongoing parallel activity. Such research interests, of course, remain motivated by needs of modern technology" [8]. Various efficient continuous and numerical methods have been developed thus far [3–5,7–27], and are under developing.

Tornabene et al. [11] provide a chronological scheme of various numerical methods, which can be categorized into the strong form methods and the weak form methods. The strong form methods include the differential quadrature method (DQM) [8–11], the local adaptive differential quadrature method (La-DQM) [12] and the discrete singular convolution (DSC) algorithm [13–21]. The weak form methods include the isogeometric approach [7], the weak form quadrature element method (QEM) [22–27], and the spectral finite element method (SFEM) [28].

The development of the DQM and its applications in structural analysis are described in detail in [8–11]. The DSC algorithm, proposed by Wei [13], is successfully used for analysis of various structural members, including beam [14], plate [15], tapered micro-columns [16], curved panel [17], and conical shell [18]. The DSC is especially suitable for high-frequency vibration analysis, since it can yield accurate lower mode frequencies as well as relatively accurate high mode frequencies at the same time [19–21]. It is seen that, however, the applications of the DQM, La-DQM and DSC for 3D structural analysis are rare.

The weak form quadrature element method (QEM) is proposed by Striz et al. [22]. After further development by several researchers, e.g., Xing and Liu [23], Zhong and Yue [24], Jin and Wang [25], now the QEM has been projected by its proponents as a potential alternative to the conventional finite element method [26]. More recently, the QEM has been successfully used for 3D vibration analysis of elastic regular solids with different boundary conditions [27].

From the literature review, it is seen that either 3D geometrically exact beam theories [1,7] or the 3D elasticity theory [5] are used for analyzing the curved and pre-twisted beams. If the 3D elasticity theory is directly used, the curved and pre-twisted beams with regular and irregular shapes are usually mapped into a regular computational domain [5]. Both approaches result into very complex governing equations, and expressions of force boundary conditions. Therefore, the weak form methods seem more suitable for analyzing such problems, since the stiffness and mass matrices can be easily obtained by using the numerical integration, and only essential boundary conditions need to be satisfied.

In this paper, a novel method, called the sub-parametric quadrature element (SP-QE) method, is developed to analyze the free vibration of curved and pre-twisted beamlike structures with regular and irregular shapes of cross-sections. The technique is an extension of the existing QEM with regular shapes [27] by mapping the curved and pre-twisted irregular solid into a regular cube. A sub-parametric solid element is then developed via the sub-parametric technique and the differential quadrature (DQ) law. Convergence studies are carried out to show the performance of the proposed method. Numerical examples of curved or/and pre-twisted beams with rectangular, circular, elliptical and airfoil shapes of cross-sections are analyzed. Three combinations of boundary conditions are considered. For verifications, the results are compared with either finite element data or 3D spectral-Tchebychev solutions. Finally some conclusions are drawn based on the presented results.

2. Sub-parametric quadrature solid element formulations

2.1. Expressions of strain energy and kinetic energy

A curved and pre-twisted beam with an irregular shape of cross-section is considered, schematically shown in Fig. 1a. Assume that the material is homogeneous and isotropic. Symbols E, G, μ, ρ represent the elasticity modulus, shear modulus, Poisson's ratio, and the mass density, symbols L and V represent the length and the volume of the beam, and symbols β and α represent the curvature and pre-twist rate, respectively.

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