Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Procedia 29 (2018) 268–278

Transportation Research Procedia

www.elsevier.com/locate/procedia

6th CEAS AIR & SPACE CONFERENCE AEROSPACE EUROPE 2017, CEAS 2017, 16-20 October 2017, Bucharest, Romania

# A method for calculus of Internal Forces

Thien Van NGUYEN[a]*, I. Stroe[a], A. Craifaleanu[a], R. Petre[a], D. Dumitriu[a,b]

[a]*POLITEHNICA University of Bucharest, No 313 Splaiul Independentei, Bucharest 060042, Romania*
[b]*Institute of Solid Mechanics of the Rmanian Academy, No 15 Constantin Mille, Bucharest 010141, Romania*

**Abstract**

A new method to compute internal forces in a multi-bodies system is presented in the paper. Lagrange equations are used to study the motion of a system under the action of known external forces. If an internal force has to be found, a supplementary mobility is considered in the system and the corresponding internal force for the new mobility is found for null value of the mobility, as well as its first and second orders of derivatives. The method is a general one, but a particular case of mechanism used in the dynamics of the airplane elevator is analyzed to verify the validity of the proposed method.


Peer-review under responsibility of the scientific committee of the 6th CEAS Air & Space Conference Aerospace Europe 2017.

*Keywords:* airplane; multi-bodies system; constraint; dynamics.

**Nomenclature**

| | |
|---|---|
| $E$ | Total kinetic energy expressed with the base frame |
| $Q_k$ | Generalized force |
| $U_\Phi$ | Analytical function |
| $\{\mathbf{J}_{Ck}\}$ | Matrix of inertia of link $k$ about link frame $C_k x_k y_k z_k$ |
| $\{\mathbf{r}_{Ck}\}$ | Position vector of mass center of link $k$ with respect to the base frame |
| $\{\dot{\mathbf{r}}_{Ck}\}$ | Derivative of position vector of mass center of link $k$ with respect to the base frame |
| $\Re_{h+1}$ | Internal force |
| $\{\boldsymbol{\omega}_k\}$ | Angular velocity vector of link $k$ with respect to the base frame |
| $\delta W_k$ | Virtual work produced by forces acting upon the system corresponding to $\delta q_k$ |

* Corresponding author. Tel.: +40-734-874-465.
*E-mail address:* bangden33468@gmail.com

2352-1465 
Peer-review under responsibility of the scientific committee of the 6th CEAS Air & Space Conference Aerospace Europe 2017.
10.1016/j.trpro.2018.02.024

| | |
|---|---|
| $\delta q_k$ | Virtual displacement |
| $\lambda_i$ | Lagrange multiplier |

## 1. Introduction

Determining internal forces and constraint force are important steps in dynamic analysis, which is the base for structure design of a mechanism. As known, calculus of internal forces for a static system of rigid bodies is common in the field known as strength of materials. According to the standard procedure, first, external reactions at external supports need to be computed and next, by using sections perpendicular to the rigid body axis, the internal forces such as shear force, or bending moment at specified points along the rigid body are calculated based on the principle of equilibrium. However, for a complex mechanism with a large number of degrees of freedom, the analysis of the constraint forces in dynamic state are extremely difficult. Consequently, the calculus of the internal forces will encounter a lot of difficulties.

In recent years, the problems related to the dynamic analysis of rigid bodies systems have attracted attention of researchers and some of them have got valuable results in their works: based on the observation method and the theory of the reciprocal screw system, (Zhi and Wang, 2015) have solved and expressed the constraint forces of the kinematic pair of a slider-crank mechanism and a single loop spatial RUSR mechanism by introducing the solution coefficient of the constraint wrench of the kinematic pair; (Y. Zhao, J. F. Liu and Z. Huang, 2011) used the screw theory to determine all the reactions, as well as the active forces, for the spatial 3-RPS parallel manipulator (consisting of a mobile platform connected to a fixed base by three identical supporting limbs with symmetrical kinematic structure) without over-constraint; using Newton-Euler method and d'Alembert's principle, (Y. Jiang, T. M. Li and L.P. Wang, 2011) established the force analysis equations and also put forward the dynamic analysis model of a parallel mechanism based on the deformation compatibility method; (A. Rotaru, L. Dudici, 2016) calculated the reaction wrench components of all the kinematic pairs of the linkages of the Stewart platform by applying the Denavit-Hartenberg transformation matrices and the principle of virtual work; by using Lagrange equations and the principle of virtual work, (I. Stroe and S. Staicu, 2010) calculated the joint forces in the double pendulum; (I. Stroe, S. Staicu and A. Craifaleanu, 2011) determined the bending moment in a compass robotic arm based on Lagrange equations.

As known, by using Lagrange equations, the differential equations of motion of a rigid bodies system can be obtained easily without considering constraint forces. In addition, if an internal force has to be found, a supplementary mobility related to it is considered in the system and the corresponding internal force for the new mobility is calculated for null values of mobility as well as its first and second derivatives. The paper presents a new method for determining internal forces. Not only this method can calculate the internal forces in a rigid body but it can also calculate them in a group of links having translational movement one with respect to the other. A system for controlling the aircraft elevator is considered to illustrate the proposed method.

## 2. Method for calculating Internal Forces by using Lagrange Equations

### 2.1. Equations of motion of a rigid bodies system

When constraints are expressed by functions of coordinates, the motion of the systems can be studied with Lagrange equations for holonomic systems with dependent variables, while if the constraints are expressed by velocities, the motion is described with Lagrange equations for non-holonomic systems.

For a non-holonomic system, the Lagrange equations corresponding to a system of $h$ generalized coordinates

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E}{\partial \dot{q}_k}\right)-\frac{\partial E}{\partial q_k}=Q_k+\sum_{i=1}^{p}\lambda_i a_{ik}\ ,\ \ (k=1,\ 2,...,h) \tag{1}$$

are completed with the constraints