ARTICLE IN PRESS

Computers and Structures xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Derivation of the out-of-plane behaviour of masonry through homogenization strategies: Micro-scale level

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ARTICLE INFO

Article history:
Received 15 April 2018
Accepted 22 August 2018
Available online xxxx

Keywords:
Masonry
Homogenization
Multi-scale
Micro-scale
RVE

ABSTRACT

Two simple and reliable homogenized models are presented for the characterization of the masonry behaviour via a representative volume element (RVE) defined at a structural level. An FE micro-modelling approach within a plate formulation assumption (Kirchhoff-Love and Mindlin-Reissner theory) using Cauchy continuum hypotheses and first-order homogenization theory is adopted. Brick units are considered elastic and modelled through quadrilateral finite elements (FEs) with linear interpolation. Mortar joints are assumed to be inelastic and reduced to zero-thickness interface FEs. A multi-surface plasticity model governs the strength envelope of mortar joints. It can reproduce fracture, frictional slip and crushing along the interface elements, hence making possible the prediction of a stepped, toothed or de-bonding failure pattern of masonry.

Validation tests on the homogenized procedures are undertaken to conclude on the correct identification of the elastic stiffness properties, in the ability to reproduce the masonry orthotropic behaviour and the effect of potential pre-compressive states. Furthermore, the approaches are extended to characterize a case study of an English-bond masonry wall. Both the validation and application steps provide excellent results when compared with available experimental and numerical data from the literature. Conclusions on the influence of three-dimensional shear stresses and the effect of potential discontinuities along the thickness direction are also outlined.

The two homogenized approaches are, for the running- and English-bond masonry cases, integrated within a FE code. By providing reliable and low computational cost solutions', these are particularly suitable to be combined within multi-scale approaches.

1. Introduction

The analysis of the masonry behaviour in terms of strength and deformation modes is still a challenge. Such complexity arises from: (i) the material heterogeneity, because of the staggering between units and mortar joints; (ii) the non-linearity of the material components; and (iii) the existence of planes of weakness which tend to govern the behaviour and damage, because mortar joints are typically less stiff and less resistant than block units [1].

Advanced computational methodologies are being developed and constitute important tools for the analysis of masonry structures [2]. Approaches such as the discrete element method are quite accurate for the study of dry or weak mortar masonry structures and examples of its application can be seen in [3,4]. These follow a large deformations formulation and with a contact updating between block units, which can be rather rigid or deformable. Yet, conducting a dynamic analysis within a 3D problem demands high processing times. Other advanced numerical strategies, such the ones based on the finite element (FE) method are still receiving more attention from the scientific community, being commonly designated as: (i) the direct simulation or the micro-modelling approach, where units and joints are represented individually; (ii) the macro-modelling approach, where masonry is represented as a homogeneous material; and (iii) the multi-scale computational approach. The reader is referred to [2] for a comprehensive overview of such strategies.

The approach proposed in this paper belongs to the so-called multi-scale methods based on the homogenization theory. Homogenization is basically an averaging procedure performed at a micro-scale upon a Representative Volume Element (RVE). On the RVE, a Boundary Value Problem (BVP) is formulated allowing an estimation of the expected average response to be used as

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https://doi.org/10.1016/j.compstruc.2018.08.013
0045-7949/

Please cite this article in press as: Silva LC et al. Derivation of the out-of-plane behaviour of masonry through homogenization strategies: Micro-scale level. Comput Struct (2018), https://doi.org/10.1016/j.compstruc.2018.08.013

constitutive relations at a macro-level. This framework has been used to investigate the behaviour of composites with different natures [5–11] but is also useful for the study of masonry structures [12–18]. Homogenization theory seems the most efficient compromise between micro- and macro-modelling. The use of such an approach is appealing because it allows deriving the macro-behaviour of masonry through the micro-scale characterization and thus considering its texture, components properties and expected micro-failure modes. In this way, the computational burden (in terms of CPU) is significantly reduced if compared with a fully micro-mechanical description of the material, as demonstrated in [19].

The multi-scale finite element computational homogenization methods, see [5,7,10,20–23], typically rely on a micro and macro transition of information and are thus designated as two-scale or FE^2 approaches. The classical models are based on a first-order homogenization scheme and, as its formulation relies on the first gradient of the kinematics field, two main limitations may arise. The first is related to the principle of separation of scales, which enforces the assumption of uniformity upon the macroscopic fields attributed to each RVE. It is known that in macro-regions where high deformation gradients are present, the latter assumption is not totally effective. The second limitation arises from the fact that the lengths of the two scales are not intrinsically considered on this classical formulation and, therefore, mesh-sensitivity issues and loss of ellipticity of the equilibrium [24] tend to appear when softening behaviour of the material is present [25]. The latter demands a regularization process, for instance upon the fracture energy terms [26,27], to guarantee the problem objectivity. In this scope, several extensions of this method were developed trying to overcome these issues. Some authors extended the classical method to a second-order homogenization [28,29], in which the constitutive behaviour is derived from both the classic part and a higher gradient part and thus, linking the length scales. Other researchers developed techniques that possibly permit the enrichment of the kinematical constraints but still allowing for the use of classical constitutive forms. This is achieved preferably through the use of Cosserat continuum models [30–32]. The well-posedness of the macroscale solution is thus achieved independently of the used mesh, even if the assumption of the separation of scales is lost.

The main advantages of the classical FE^2 approaches are two-fold: (i) flexibility on the method to be used at a micro-scale, which can be based on the FE-method [10], Fourier series [33,34], on Voronoi method [20] among others; (ii) it does not require any macro-constitutive relation, because the macro-behaviour is totally dependent on the homogenized response derived on the foregoing scale. Nevertheless, the classical FE^2 approaches (in particular the full continuum-FE methods) are still a challenge in the non-linear range [19,25]. The advantages are especially obvious when linear elastic behaviour is assumed but obtaining a micro-scale solution at each load step for each Gauss point may turn the problem prohibitive from a computational point of view. These strategies still have a higher computational cost if compared with a macro-modelling one. So, the authors believe that if one intends to use homogenization strategies for the study of large or more complex structures, the development of techniques to speed up the processing running times is critical.

Some assumptions may be undertaken which can significantly reduce the computational cost of an FE^2 approach. The use of homogenization methods based on the unit-cell theory, first proposed in the elastic range by Hashin & Rosen [35] and in the nonlinear range by Teply & Dvorak [36] through the use of the so-called hexagonal array model, is a possibility. In these methods (see [37]), closed-form expressions are derived at a micro-scale from both equilibrium and compatibility conditions at the RVE. After being solved or formulated these can provide the homogenized quantities or describe phenomenologically the constitutive equations at a macro-scale, see [17,38,39]. The use of closed-formed solutions is, however, not so feasible in the non-linear range, in complex loading cases or in cases where geometrical and physical changes can occur. Another strategy is the use of the so-called adaptive multi-scale methods [40–42], which take advantage of the best of the first-order theory and micro-modelling approach. A first-order homogenized model represents initially the masonry behaviour until a threshold criterion is reached. Such criterion may be able to account for the onset of damage propagation or another high-gradient source. After reaching the threshold, the area of interest is replaced and kept by an explicit microstructural description able to represent the high localized deformation without the ill-posedness of the first-order theory, see [42] for the masonry field application. These numerical models could be a valuable tool due to its computational attractiveness. Many current studies on unreinforced masonry focus on in-plane cases and for quasi-static loading of running-bond masonry and, therefore, more research is required on structural models with other masonry texture and loading conditions, as out-of-plane loads or seismic excitations.

Besides the assumptions undertaken at a micro-scale, there is also the possibility of using simplified but still accurate methods that can be implemented at a macro-scale. The integration of these models within a micro- to macro- homogenized formulation, i.e. where the material constitutive information is transferred in one step from the micro- to the macro-scale, can be very promising especially for the dynamic study of masonry structures. In fact, some proposals can be found in the literature, for instance, the use of limit analysis [43], or the use of discontinuous or discrete FE-models instead of the classical macroscale continuum-FE strategies. Several works demonstrate its accuracy and computational efficiency when applied to in-plane [43] and out-of-plane loaded masonry [31,44–46] but, as well, for masonry structures subjected to dynamic loads [27,47]. The application of these methods is questionable in cases where multiphase couplings may occur, as when thermal or hydro-mechanical effects may exist. Still, the latter can be disregarded to occur in structural oriented problems.

From the above considerations, the general aim of the present study is to formulate two unit-cell homogenized models. For the sake of avoiding a full three-dimensional discretization of the masonry, both homogenized strategies follow plate (but different) element formulations. Its validation is conducted considering experimental and numerical data available in the literature oriented for both in- and out-of-plane analysis of unreinforced periodic masonry structures which may be linked with a proper macro-scale model.

The majority of the existing research on masonry deals with running-bond texture within a single-wythe walls case [12,17,18,39,48–50], being the study of English-bond textures somehow under-investigated [47,51]. The novelty of this work is to present two homogenized-based models oriented for both in- and out-of-plane analysis of English-bond masonry structures. Due to its formulation differences, conclusions on the influence of three-dimensional shear stresses and the effect of discontinuities/transversal joints along the masonry thickness can be drawn. In the analysis, both linear and non-linear ranges are accounted, in which masonry orthotropy and full softening behaviour are reproduced (material nonlinearity lumped on mortar joints).

At last, it may be addressed that the procedures are fully integrated within the commercial software DIANA [52] by exploiting its programming features. These are ready to be combined with a FE^2 approach but, noticing the raised issues of full FE-continuum homogenized strategies, especially suitable to be linked with a discrete-FE macro model aiming to obtain reliable results with a quite attractive computational cost.

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