Computers and Structures xxx (2018) xxx–xxx

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Computing tight bounds of structural reliability under imprecise probabilistic information

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A R T I C L E   I N F O

Article history:
Received 21 March 2018
Accepted 19 July 2018
Available online xxxx

Keywords:
Structural reliability analysis
Uncertainty
Probability box
Monte Carlo simulation
Interval analysis
Imprecise probability

A B S T R A C T

In probabilistic analyses and structural reliability assessments, it is often difficult or infeasible to reliably identify the proper probabilistic models for the uncertain variables due to limited supporting databases, e.g., limited observed samples or physics-based inference. To address this difficulty, a probability-bounding approach can be utilized to model such imprecise probabilistic information, i.e., considering the bounds of the (unknown) distribution function rather than postulating a single, precisely specified distribution function. Consequently, one can only estimate the bounds of the structural reliability instead of a point estimate. Current simulation technologies, however, sacrifice precision of the bound estimate in return for numerical efficiency through numerical simplifications. Hence, they produce overly conservative results in many practical cases. This paper proposes a linear programming-based method to perform reliability assessments subjected to imprecisely known random variables. The method computes the tight bounds of structural failure probability directly without the need of constructing the probability bounds of the input random variables. The method can further be used to construct the best-possible bounds for the distribution function of a random variable with incomplete statistical information.

1. Introduction

The various sources of uncertainties arising from structural capacities and applied loads, as well as computational models, are at the root of the structural safety problem of civil structures. In an attempt to measure the safety of a structure, it is necessary to quantify and model these uncertainties with a probabilistic approach so as to further determine the failure probability [1–4]. In a reliability assessment, the identification of the probability distributions of the random variables is crucial. The uncertainty associated with a random variable can be classified into either aleatory or epistemic [5], with the former arising from the inherent random nature of the quantity, and the latter due to knowledge-based factors such as imperfect modelling and simplifications, and/or limited supporting database. Statistical uncertainty is an important source of the epistemic uncertainty, which accounts for the difference between the probability model of a random variable inferred from limited sampled data and the “true” one. This uncertainty may be significant if the size of available data/observations is limited. To better assess the safety of a structure, structural reliability assessment needs to consider both aleatory and epistemic uncertainties [5–9].

The result of a structural reliability assessment may be sensitive to the selection of the probability distributions of the random inputs [10]. However, in many cases, the identification of a variable’s distribution function is difficult or even impossible due to limited information/data. Rather, only incomplete information such as the first- and the second- order moments (mean and variance) of the variable can be reasonably estimated. In such a case, the incompletely-informed random variable can be quantified by a *family* of candidate probability distributions rather than a single known distribution function. This is the basic concept of *imprecise probability* [11]. As a result, the structural reliability in the presence of incompletely-informed random variables can no longer be uniquely determined. A practical way to represent an imprecise probability is to use a probability bounding approach by considering the lower and upper bounds of the imprecise probability functions. Under this context, approaches of interval estimate of reliability have been used to deal with reliability problems with imprecise probabilistic information [12], including the

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https://doi.org/10.1016/j.compstruc.2018.07.003
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probability-box (p-box for short) method [13], random set and Dempster-Shafer evidence theory [14–16], fuzzy random variables [17], and others. These methods are closely related to each other, and may often be used as equivalent for the purpose of reliability assessment [13,18]. However, the bounds of structural reliability estimated using a probability bounding approach may be overly conservative in some cases, due to the fact that it only considers the bounds of the distribution function, thus some useful information inside the bounds may be lost. This fact calls for an improved approach for reliability bound estimate which can take full use of the imprecise information of the variable(s).

Over the last decade many efforts have been directed towards structural reliability assessment using imprecise probability theory. In [19], random variables and interval variables are considered simultaneously. Monte Carlo simulation was used with function approximation to reduce the total number of simulations. In [13,20], imprecisely probability distribution functions were modeled using probability-boxes and Dempster-Shafer structures. The reliability analysis was based on the Cartesian product method and interval arithmetic. The framework was applied to environmental risk assessment. Schweiger and Peschl [21] considered stochastic finite element analyses of a deep excavation problem in which the uncertain material parameters and geometrical data were modeled as random sets. The random sets were propagated through the finite element analysis using the vertex method, under the assumption that the structural response is monotonic with respect to each random set variable. In [22], structural reliability evaluations in the presence of both random variables and interval variables were considered. The limit state functions were approximated using the response surface method to reduce the computational cost. In [23], the Tchebycheff's inequality was proposed to construct random set models of a random variable using the information of mean and standard deviation. The approach was demonstrated using two geotechnical problems. An interval Monte Carlo method was developed in [9] for structural reliability assessment under epistemic uncertainties. An imprecise cumulative distribution function with interval parameters is modeled as a probability-box. In each simulation, interval-valued samples are sampled and the range of the limit state function is computed using interval analysis. A similar approach, namely the unified interval stochastic sampling approach, was proposed in [24] to determine the statistics of the lower and upper bounds of the collapse loads of a structure involving mixture of random and interval parameters. Variance-reduction techniques have been proposed to combine with the interval Monte Carlo simulation to enhance the computational efficiency, e.g., the interval importance sampling technique [18], the interval Quasi-Monte Carlo sampling [25], and subset sampling [16,26].

Mathematically, the use of the (complete) moment information of a random variable is equivalent to its probability distribution function since knowing one can determine the other completely through the moment generation function [27,28]. Many previous studies have conducted reliability analysis by making use of the moment information of random variables. For instance, a second-order reliability analysis method based on an approximating paraboloid was proposed in [29]. In [30], a method for system reliability analysis was developed taking into account the moments of the system limit state function derived from point estimates. Zhao et al. [31] discussed the suitability and the monotonicity of the fourth-moment normal transformation in reliability assessment considering imprecise random inputs. Wang et al. [32] proposed an approach to estimate the time-dependent reliability of aging structures in the presence of incomplete deterioration information. The motivation of using (limited) moment information for reliability assessment is due to the fact that in many cases only limited observations/samples of a random variable are accessible, and thus the estimation of the moments (typically the low order moments such as mean and variance) based on the limited samples is relatively straightforward and more reliable as compared with estimating the complete distribution function. One illustrative case of imprecise probability information is that the standard deviation of a random variable, say, X, can be roughly estimated from limited observations by 0.25 times the difference between the maxima and minima of the samples, if the observed values are believed to vary with a range of mean$\pm 2\times$standard deviation of X; however, the determination of the specific distribution type of X depends on further probabilistic information. In the presence of the raw data only, Zhang and Shields [33,34] employed the multiple probability models that include different model families with uncertainties associated with the model parameters to propagate the imprecise probability information. This paper considers the case of reliability assessment with imprecise probabilities in which only the low-order moments of a random variable are known, while the distribution type and distribution function are unknown.

This paper proposes a linear programming-based method for solving the reliability problems in the presence of imprecise probabilistic information. The estimate of reliability bounds is transformed into finding the solution of a linear objective function, where the constraint equations are established by taking full use of the information of moments, and the range information of the random variable if available. Two types of objective functions are developed independently, which can verify the accuracy of the solutions mutually, and provide insights into the problem from different perspectives. The paper first introduces the methodology for the problems involving only one imprecise random variable; then an iterative approach is proposed to handle the problems with multiple imprecise random variables. While the proposed method computes bounds of failure probabilities directly without first constructing the probability-boxes of the imprecisely known random input variables, it can also be used to construct the best-possible cumulative distribution function (CDF) bounds for a random variable with limited statistical information. Three examples are presented to demonstrate the application of the proposed method on these two aspects.

2. Probability-box method in the presence of imprecise random variables

2.1. Impact of imprecision on reliability assessment

A typical structural reliability problem takes the form of

$$P_f = \Pr(G(\mathbf{X}) \leqslant 0) = \int \cdots \int_{G(\mathbf{x})\leqslant 0} f_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{1}$$

where Pr denotes the probability of the event in the bracket, P_f represents the failure probability of the structure, G is the limit state function in the presence of m random inputs $\mathbf{X} = \{X_1, X_2, \ldots X_m\}$, which defines structural failure if $G < 0$ and the survival of the structure otherwise, and $f_{\mathbf{X}}(\mathbf{X})$ is the joint probability density function (PDF) of $\mathbf{X}$. The failure probability in Eq. (1) is often estimated by the well-known Monte Carlo method,

$$P_f \approx \frac{1}{N}\sum_{j=1}^{N} \mathbb{I}\left[G(\mathbf{x}_j) \leqslant 0\right] \tag{2}$$

where N is the number of replications, $\mathbb{I}[\cdot]$ is an indicator function, which returns 1 if the statement in the bracket is true and 0 otherwise, and $\mathbf{x}_j$ is the jth simulated sample of $\mathbf{X}$. $\mathbf{x}_j$ can be generated using the inverse transform method,

$$\mathbf{x}_j = F_{\mathbf{X}}^{-1}(\mathbf{r}_j), \quad j = 1, 2, \ldots, N \tag{3}$$

Please cite this article in press as: Wang C et al. Computing tight bounds of structural reliability under imprecise probabilistic information. Comput Struct (2018), https://doi.org/10.1016/j.compstruc.2018.07.003