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On virtual pathwidth of virtual graphs of a virtual link

Noureen A. Khan

University of North Texas at Dallas, United States

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ABSTRACT

The geometric representation of a knot is not too dissimilar from a graph and this interaction has helped mathematicians to solve many problems. In this paper, we apply graph theory tools to study the classification of virtual knots and links. We define virtual planar graphs and compute virtual path width of an associated graph of a virtual link. We show that the virtual path width of an associated graph is equal to the virtual bridge number of a pseudo prime knot.

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1. Introduction

Topological and planarity properties of a graph, sometimes, make it more convenient to study the graph of a link than the link itself. Over the years, the interactions between graphs and links has benefited mathematicians to solve many problems in both directions. In this paper, we use the techniques from graph theory to study the invariants of virtual knots and links. In particular, we define virtual pathwidth of a graph associated to a virtual links and its relationship with the virtual bridge number. We consider a virtual knot diagram as a discrete geometric object more than its exclusive topological description when defining the corresponding notions by graph theory tools. We begin this section with a brief review of virtual knots and links intrinsic to define virtual planar graphs.

Introduced by Louis Kauffman, the theory of virtual knots is based on the knots in thickened surfaces. A virtual knot represents a natural combinatorial generalization of a classical knot, [4]. A virtual knot diagram corresponds to a unique embedding in a thickened surface of minimal genus. One obtains a *virtual knot diagram* by first projecting the knot on to a plane (S^2) and then slightly changing the double points to over and under crossings, as they appear on the knot. So, a knot diagram is a closed curve in S^2, and clearly, many diagrams can represent the same knot. In addition to over and under crossings, on a virtual knot diagram, one also obtains *virtual crossings*, that are neither over nor under, just artifact of the knot projection. A virtual crossing should be considered as a diagrammatic picture of two parts of a knot on the plan which are far from each other, the strands do not actually cross in real but appear on the knot

E-mail address: noureen.khan@unt.edu.

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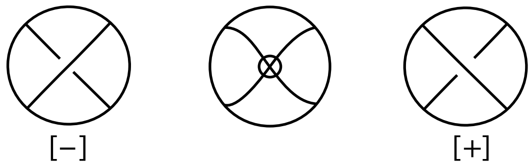

Fig. 1. Negative, virtual and positive crossings.

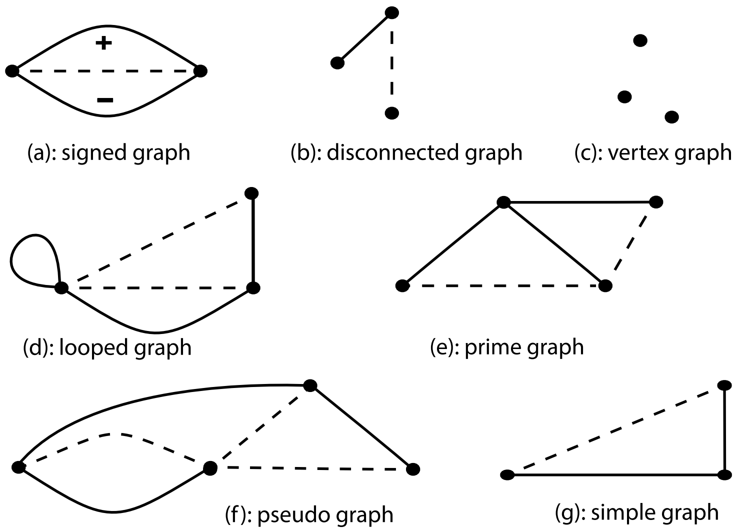

Fig. 2. The basic virtual graphs.

projection, [4]. In other words, to draw a virtual knot diagram, one needs to know only the position of real crossing points and how they are connected to each other; whereas, the position of paths connecting real crossings, intersections and self-intersections does not matter. Any two virtual knot diagrams are equivalent if one can be transformed into the other by a finite series of classical and virtual Reidemeister moves [4]. Therefore, a virtual link diagram has real crossing points, either negative or positive, and virtual crossing points as illustrated in Fig. 1. In general, when dealing with a virtual link, we assign orientation to each component, and this orientation automatically induces the orientation to the link projection. This work primarily focus on the virtual knots, rather digging into the technical discussions that involve orientation and rigor mathematical existence of virtual links. Moreover, all diagrams are assumed to be reduced and oriented, unless specified otherwise.

1.1. Planar graph of a virtual knot

A *planar graph* of a knot diagram is a finite set of points (*vertices*), connected by segments (*edges*), which meet only at the vertices. To define a virtual planar graph, in addition to the real edges, one also needs *virtual edges* to represent virtual crossings of the virtual link diagram. We say, a *virtual planar graph* G_v is a set of finite points (*vertices*) connected by real or virtual segments (*edges*) that meet only at the vertices. The sets of vertices and edges (real and virtual) are denoted by $V(G_v)$ and $E(G_v)$ respectively. Since a virtual link diagram has finite number of crossings, both $V(G_v)$ and $E(G_v)$ are finite sets. Therefore, a virtual planar graph G_v is always a finite set. On a virtual planar graph, we use dotted segments to represent virtual edges to distinguish them from real edges (solid segments), see Fig. 2. Hereafter, we will a virtual planar graph as a *virtual graph.*

A *signed virtual graph* G_v carries additional information, such as *positive* (+) or *negative* (−) signs assigned to real edges, however, its virtual edges remain unsigned, see Fig. 2 (a). A *looped virtual graph* is