Applied Mathematics and Computation 340 (2019) 1–4

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Infinite family of 2-connected transmission irregular graphs

Andrey A. Dobrynin[a,b,*]

[a] Novosibirsk State University, Novosibirsk 630090, Russia
[b] Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk 630090, Russia

ARTICLE INFO

MSC:
05C12
05C05
05C35

Keywords:
Vertex transmission
Transmission irregular graph
Wiener complexity

ABSTRACT

Distance between two vertices is the number of edges in the shortest path connecting them in a connected graph G. The transmission of a vertex v is the sum of distances from v to all the other vertices of G. If transmissions of all vertices are mutually distinct, then G is a transmission irregular graph. It is known that almost no graphs are transmission irregular. Infinite families of transmission irregular trees were presented in [4]. The following problem was posed in [4]: do there exist infinite families of 2-connected transmission irregular graphs? In this paper, an infinite family of such graphs is constructed.

1. Introduction

All graphs considered in this paper are undirected, connected, without loops and multiple edges. The vertex set of a graph G is denoted by $V(G)$. Under distance $d(u, v)$ between vertices $u, v \in V(G)$ we mean the standard distance of a simple graph G, i. e., the number of edges on a shortest path connecting these vertices in G. The transmission, $\mathrm{Tr}(v)$, of vertex $v \in V(G)$ is defined as the sum of distances from v to all the other vertices of G. A half of the sum of vertex transmissions gives the Wiener index that has found important applications in chemistry (see books [7,12–14,20] and selected papers [6,8–10,15–19]). Transmissions of vertices are used for design of distance-based information topological indices [5]. The number of different vertex transmissions is known as the Wiener complexity of a graph [1–3]. A graph is called transmission irregular if it has the largest possible Wiener complexity over all graphs of a given order, i. e., vertices of the graph have pairwise different transmissions.

It was shown that almost all graphs are not transmission irregular [4]. This follows from the facts that almost every graph has diameter 2 and a graph with diameter 2 is not transmission irregular. Transmission irregular graphs and their Wiener index were studies in [4]. In particular, infinite families of transmission irregular trees were described and the following problem was formulated: do there exist infinite families of 2-connected transmission irregular graphs? In this paper, an infinite family of such graphs is constructed.

2. Infinite family of transmission irregular graphs

Transmission irregular graphs of small order without cut-vertices can be found by computer calculations. Smallest graphs of this class have order 9 (see Fig. 1). There are 25 and 817 transmission irregular 2-connected graphs of order 10 and 11, respectively (see Fig. 2). The number of non-planar graphs of order 11 among them is 161. The first graph in the sixth row of

* Corresponding author at: Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk 630090, Russia.
E-mail address: dobr@math.nsc.ru

https://doi.org/10.1016/j.amc.2018.08.042

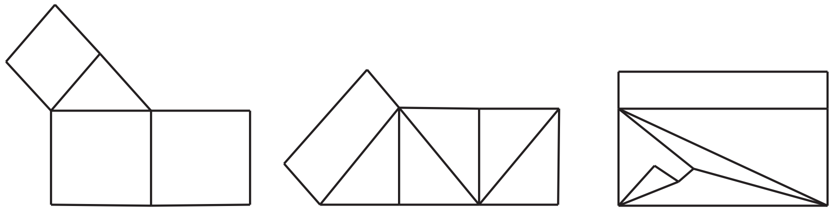

Fig. 1. 2-connected transmission irregular graphs of order 9.

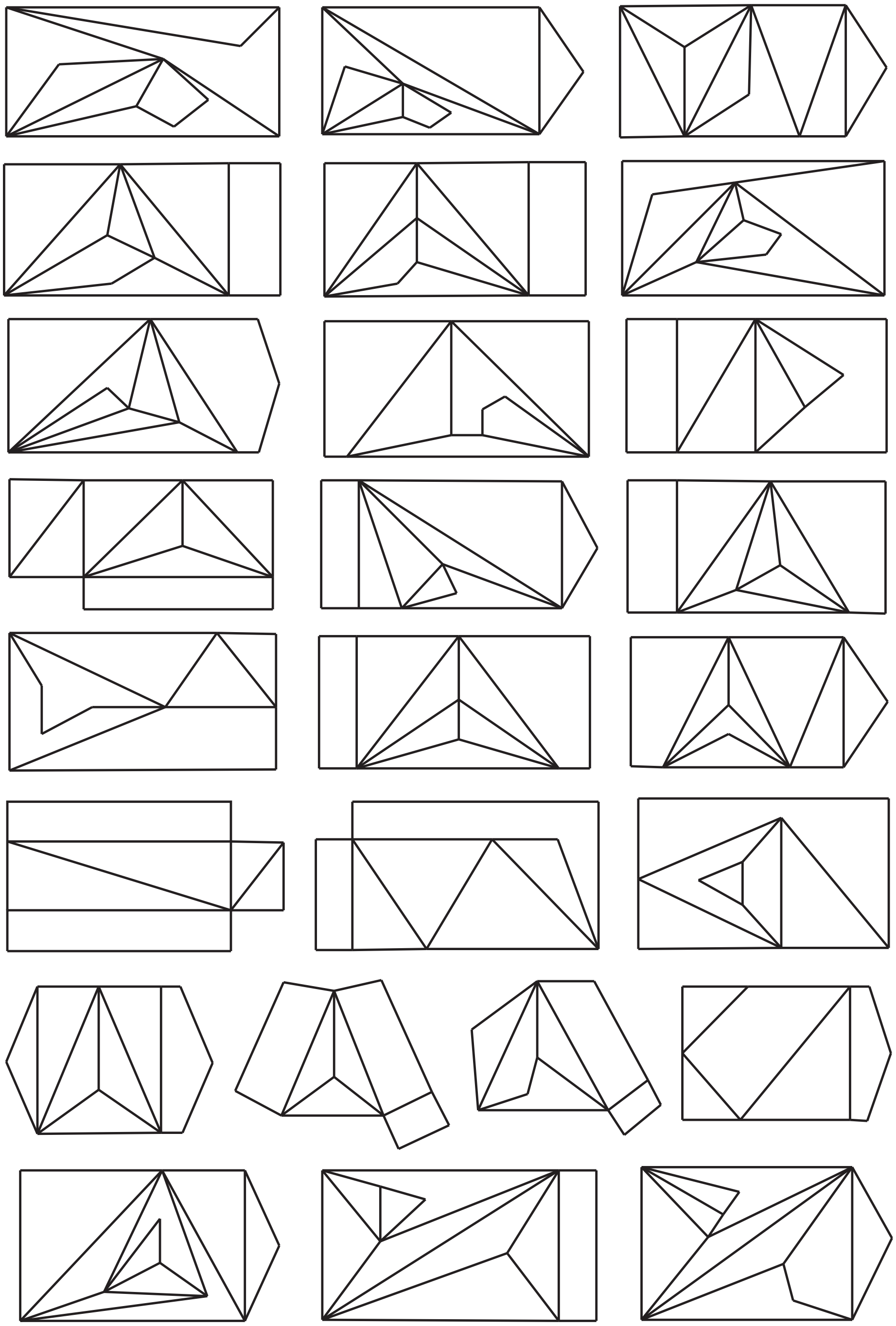

Fig. 2. 2-connected transmission irregular graphs of order 10.