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Model reference tracking control for spatially interconnected discrete-time systems with interconnected chains☆

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ABSTRACT

This study is concerned with the model reference tracking control problem for spatially interconnected systems (SISs) with interconnected chains in discrete-time case. The aim of this paper is to design a state-feedback controller such that the state of the controlled system tracks precisely a given stable reference signal or the output of the controlled system tracks the output of a given reference model well. First, two sufficient conditions are given to guarantee that the closed-loop system is asymptotically stable and the corresponding tracking performance requirement is satisfied, respectively. Next, the existence of the desired state-feedback controllers are obtained by the proposed methods based on linear matrix inequalities. Finally, a first-order partial differential equation simulation example is provided to illustrate the effectiveness of the derived methods.

1. Introduction

Recently, research on control theory of linear and nonlinear systems develops rapidly [1–8]. In this paper, spatially interconnected systems (SISs) with interconnected chains in discrete-time case are considered. SISs consist of similar subsystems which interact with their closest neighbors [9,10]. During the last few decades, SISs have broad applications in power systems, automated highways systems [11,12], airplane formation flight [13], satellite formations [14,15], cross-directional control in paper processing [16], partial differential equation, and other areas [17–19].

Since the complicated behaviors and wide applications, SISs have been studied by many scholars in recent years. The basic conceptions of SISs and the temporal and spatial forward operators for SISs are introduced in [9,10]. Meanwhile, the analysis, synthesis, and implementation of distributed controllers for SISs are successfully tackled. There are other results on multidimensional filters for SISs [20], iterative learning control for SISs [21], spatially invariant distributed dynamical systems [22], the systems interconnected over an arbitrary graph [23], SISs with time delay [24–26], a scaled small gain theorem for SISs [27], SISs with structured uncertainties [26,28–31], etc.. Specially, SISs with interconnected chains are investigated in [18,19,32]. The close formation flight is modeled as SISs with interconnected chains and the corresponding distributed control design problem is solved in [18]. The distributed parameter dependent controller is proposed for parameter varying SISs with interconnected chains in [19]. The reference [32] presents a 2-D hybrid Roessor model for SISs with interconnected chains in one spatial dimension and develops the corresponding Kalman-Yakubivich-Popov lemma for derived model.

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State/output tracking is an important issue for system and control theory, since its wide applications in robot control, flight control and other practical fields [33–35]. The objective of state tracking control problem is to design a state-feedback controller such that the state of the controlled system follows a given stable reference signal well. The goal of output tracking control problem is to design a state-feedback controller such that the output of the controlled plant tracks the output of a given reference signal as close as possible. Over the last two decades, many scholars have been concentrated on state/output tracking control design, and many methods have been introduced to solve this control problem [36–47]. It is worth to be pointed that model reference tracking control design is more general and more difficult than the stabilization problem, since model reference control design requires that the controlled system not only to be stabilized, but also to meet a specified tracking performance [35,48,49]. Thus, the model reference tracking problem for SISs with interconnected chains is a significant problem. To the best of authors' knowledge, this problem has not been investigated yet, which motivates our study.

In this paper, we focus on the problem of model reference tracking control for SISs with interconnected chains in discrete-time case. First, two sufficient conditions are derived to guarantee the asymptotic stability and the tracking performance of the closed-loop systems. Then, the most important contribution of this paper is the design of expected state-feedback controllers which achieve the specified tracking performance. Thus, the model reference tracking problem for complicated SISs with interconnected chains can be successfully solved by applying designed state-feedback controllers.

The paper is organized as follows. The preliminaries and state tracking control design are given in Section 2. The output tracking control design is presented in Section 3. A first-order partial differential equation example is demonstrated in Section 4. Conclusions are given in Section 5.

Notations: The set of integers is denoted as $\mathcal{Z}$. The set of nonnegative integers is denoted as $\mathcal{Z}^+$. The set of real numbers is denoted as $\mathcal{R}$. The notation $x \in \mathcal{R}^\bullet$ is used to denote real valued, finite vectors whose size is either clear from context or not relevant to the discussion. $\mathcal{R}^{n\times m}$ denotes the space of n by m matrices; $\mathcal{R}_S^{n\times n}$ denotes the space of symmetric n by n matrices. I denotes the identity matrix when the dimension is clear from the text. Given real symmetric matrix M, $M > 0(\geq 0)$ denotes property $v^* M v > 0(\geq 0)$ for all $v \neq 0$. The notation (x_1, x_2) is used to denote the vector with two (not necessarily scalar) components. $\|\cdot\|_2$ denotes the Euclidean norm on vectors and the corresponding inner product is defined as $\langle x(k,s), y(k,s)\rangle_2 = x^*(k,s)y(k,s)$.

2. State tracking control design

2.1. Problem formulation and preliminaries

In this paper, signals are denoted by $L+1$ independent variables: $x = x(k, s_1, \ldots, s_L)$, where the temporal variable and the spatial variables are denoted by $k \in \mathcal{Z}^+$ and $s = (s_1, s_2, \ldots, s_L), s_i \in \mathcal{Z}$, respectively. The temporal and spatial variables of a signal can be clearly separated when the signals are considered at a fixed time k, which motivates the following definition.

Definition 1. [9,10] The space ℓ_2 is the set of functions mapping $\mathcal{Z} \times \cdots \times \mathcal{Z}$ to $\mathcal{R}^\bullet$ for which the following quantity is finite:

$$\sum_{s_1 \in \mathcal{Z}} \cdots \sum_{s_L \in \mathcal{Z}} x^*(s)x(s) < \infty.$$

The inner product on ℓ_2 is defined as

$$\langle x, y\rangle_{\ell_2} := \sum_{s_1 \in \mathcal{Z}} \cdots \sum_{s_L \in \mathcal{Z}} x^*(s)y(s),$$

with corresponding norm $\|x\|_{\ell_2} := \sqrt{\langle x, x\rangle_{\ell_2}}$.

Note that, for given k and s, $u(k, s)$ denotes a real-valued vector and $u(k)$ denotes an element of ℓ_2 .

In this section, we consider the following SISs with interconnected chains in discrete-time case:

$$\begin{bmatrix} (\mathbf{T}x)(k,s) \\ w(k,s) \end{bmatrix} = \begin{bmatrix} A_{TT} & A_{TS} & B_{Tu} & B_{Td} \\ A_{ST} & A_{SS} & B_{Su} & B_{Sd} \end{bmatrix} \begin{bmatrix} x(k,s) \\ v(k,s) \\ u(k,s) \\ d(k,s) \end{bmatrix}, \tag{1}$$
$$w(k,s) = (\Delta_{S,m} v(k))(s),$$

with the initial state

$$x(0,s) = x_0(s), \;\; v(k,0) = 0,$$

where $x(k,s) \in \mathcal{R}^{m_0}$ is the state variable, $u(k,s) \in \mathcal{R}^{n_u}$ is the control signal, $d(k,s) \in \mathcal{R}^{n_d}$ is the exogenous disturbance. For a given $m = (m_0, m_1, m_2, \ldots, m_L)$, the structured operator $\Delta_{S,\,m}$ is defined as:

$$\Delta_{S,m} := diag(\mathbf{S}_1 I_{m_1}, \mathbf{S}_2 I_{m_2}, \ldots, \mathbf{S}_L I_{m_L}),$$