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Pathways to fraction learning: Numerical abilities mediate the relation between early cognitive competencies and later fraction knowledge

Ai Ye, Ilyse Resnick, Nicole Hansen [1], Jessica Rodrigues, Luke Rinne, Nancy C. Jordan *

School of Education, University of Delaware, Newark, DE 19716, USA

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ABSTRACT

The current study investigated the mediating role of number-related skills in the developmental relationship between early cognitive competencies and later fraction knowledge using structural equation modeling. Fifth-grade numerical skills (i.e., whole number line estimation, non-symbolic proportional reasoning, multiplication, and long division skills) mapped onto two distinct factors: *magnitude reasoning* and *calculation*. Controlling for participants' ($N = 536$) demographic characteristics, these two factors fully mediated relationships between third-grade general cognitive competencies (attentive behavior, verbal and nonverbal intellectual abilities, and working memory) and sixth-grade fraction knowledge (concepts and procedures combined). However, specific developmental pathways differed by type of fraction knowledge. Magnitude reasoning ability fully mediated paths from all four cognitive competencies to knowledge of fraction concepts, whereas calculation ability fully mediated paths from attentive behavior and verbal ability to knowledge of fraction procedures (all with medium to large effect sizes). These findings suggest that there are partly overlapping, yet distinct, developmental pathways from cognitive competencies to general fraction knowledge, fraction concepts, and fraction procedures.

* Corresponding author.
E-mail address: njordan@udel.edu (N.C. Jordan).
[1] Current address: School of Education, Fairleigh Dickinson University, Teaneck, NJ 07666, USA.

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Introduction

Fractions are a crucial topic in mathematics education. Fraction knowledge is critical for learning more advanced mathematics such as algebra (e.g., Bailey, Hoard, Nugent, & Geary, 2012; Booth & Newton, 2012; National Mathematics Advisory Panel [NMAP], 2008; Siegler et al., 2012), which in turn supports success in science, technology, engineering, and mathematics (STEM) disciplines (Sadler & Tai, 2007). Unfortunately, fractions are hard for many students (e.g., Hecht, Close, & Santisi, 2003; NMAP, 2008; Siegler et al., 2012; Vamvakoussi & Vosniadou, 2010) and often continue to be a stumbling block after formal fractions instruction has been completed (Hansen, Jordan, & Carrique, 2015). For example, on a recent assessment, only 41% of eighth graders could successfully solve a multi-step problem involving fractions (National Assessment of Educational Progress [NAEP], 2013). Difficulties with fractions often continue into adulthood. Approximately one third of a sample of U.S. community college students could not identify which of two fractions had the larger value despite being highly successful at such comparisons with whole numbers (Schneider & Siegler, 2010).

Although fraction learning has recently attracted attention among researchers in the learning and cognitive sciences (e.g., Fuchs et al., 2013; Jordan et al., 2013; Vukovic et al., 2014), the processes underlying the development of fraction knowledge are still not well understood. A complete theory of fraction development must consider not just global relationships between early predictors and later fraction knowledge but also both direct and indirect pathways to acquisition of fraction knowledge. Some competencies may be *direct* precursors of fraction knowledge; for example, the ability to represent whole numbers on a number line could directly influence the mental structure for seeing how fraction magnitudes are interspersed between numbers (Siegler, Thompson, & Schneider, 2011). Other more general cognitive abilities may *indirectly* affect fraction learning via such direct precursors of fraction understanding, with the precursor serving as an intermediary. For example, verbal intelligence (a general cognitive competency) may support the acquisition of whole number knowledge (a direct precursor), which then supports the acquisition of fraction knowledge (Vukovic et al., 2014). Furthermore, it is important to consider how these direct and indirect pathways support different aspects of fraction knowledge, which includes both conceptual and procedural knowledge.

In the current longitudinal study, we sought to clarify key developmental pathways from more general cognitive processes in third grade and precursor numerical skills in fifth grade to different types of fraction knowledge in sixth grade.

Defining fraction knowledge

Mathematical competency in any domain involves both concepts and procedures (Geary, 2004). Fraction concepts require the ability to represent fractions as magnitudes, which allows students to compare and order fractions based on their size (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010; Siegler, Fazio, Bailey, & Zhou, 2013). Skill with fraction procedures, on the other hand, involves performing arithmetic operations on fractions (Siegler et al., 2013).

Previous research reveals the importance of constructing not only a general model of fraction knowledge but also independent developmental models for concepts and fraction procedures. To be sure, fraction concepts and procedures are closely related and build on one another (e.g., Hecht et al., 2003; Hecht & Vagi, 2010; Hecht & Vagi, 2012). However, general cognitive and number-related abilities differentially predict the development of fraction concepts and procedures, respectively (Hansen, Jordan, Siegler et al., 2015; Jordan et al., 2013; Namkung & Fuchs, 2016). Each type of knowledge follows a relatively distinct pattern of growth (Hansen, Jordan, & Rodrigues, 2015), with fraction concepts being particularly important to later mathematics achievement (Hallett, Nunes, & Bryant, 2010; Hecht et al., 2003; Rittle-Johnson, Siegler, & Alibali, 2001). Thus, there is a clear need for research that considers both the broad development of fraction knowledge *and* the relationships between cognitive competencies and conceptual and procedural fraction knowledge specifically while