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Cognitive and numerosity predictors of mathematical skills in middle school

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ABSTRACT

There is a strong research base on the underlying concomitants of early developing math skills. Fewer studies have focused on later developing skills. Here, we focused on direct and indirect contributions of cognitive measures (e.g., language, spatial skills, working memory) and numerosity measures, as well as arithmetic proficiency, on key outcomes of fraction performance, proportional reasoning, and broad mathematics achievement at sixth grade ($N = 162$) via path analysis. We expected a hierarchy of skill development, with predominantly indirect effects of cognitive factors via number and arithmetic. Results controlling for age showed that the combination of cognitive, number, and arithmetic variables cumulatively accounted for 38% to 44% of the variance in fractions, proportional reasoning, and broad mathematics. There was consistency across outcomes, with more proximal skills providing direct effects and with the effects of cognitive skills being mediated by number and by more proximal skills. Results support a hierarchical progression from domain-general cognitive processes through numerosity and arithmetic skills to proportional reasoning to broad mathematics achievement.

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Introduction

Mathematical skills are important for successful living in modern society (Bynner, 2012; Gates, 2009; Geary, 2013; Kilpatrick, Swafford, & Findell, 2001; Parsons & Bynner, 2008; Rakes, Valentine, McGatha, & Ronau, 2010; Rose & Betts, 2004; Vogel, 2008). The past 20 years have seen a rapidly expanding literature on the predictors of mathematical skills across ages and types of skill (e.g., Aunio & Niemivirta, 2010; Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Cirino, 2011; Cirino, Morris, & Morris, 2002; Fuchs, Geary, Fuchs, Compton, & Hamlett, 2014; Fuchs et al., 2005, 2008, 2010a, 2010b; Geary, 1993; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Halberda, Mazzocco, & Feigenson, 2008; Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Koponen, Aunola, Ahonen, & Nurmi, 2007; Krajewski & Schneider, 2009; Mazzocco, Feigenson, & Halberda, 2011; Siegler & Ramani, 2009; Siegler et al., 2012). Much of that work has focused on the prediction of early developing mathematical skills (through Grade 3) and has been spurred by the development of influential models relevant to this stage of development (e.g., Butterworth, 1999, 2005; Dehaene, 2001; Dehaene & Cohen, 1995; Feigenson, Dehaene, & Spelke, 2004; Geary, 1993, 2004, 2013; LeFevre et al., 2010; von Aster & Shalev, 2007). These empirical and theoretical contributions have yielded strong insights into the relative contribution of differing types of predictors for the development of primary-grade math skill. Such studies set the stage for research on later developing mathematical skills given the hierarchical nature of mathematics skill learning (e.g., Fuchs et al., 2006; National Mathematics Advisory Panel [NMAP], 2008).

Although few studies have focused on more advanced mathematical skills, there have been notable advances concerning the predictors of fraction performance at later elementary grades (e.g., Hecht & Vagi, 2010; Jordan et al., 2013; Vukovic et al., 2014). The current study sought to contribute to this growing literature by clarifying the roles of individual differences from most distal to most proximal, including cognitive processes (e.g., working memory, language), numerosity (magnitude and estimation), and whole number arithmetic skills, for middle school mathematical competencies. These include fraction performance and proportional reasoning, which are themselves critical influences for later developing skills such as algebra (Hecht, Vagi, & Torgesen, 2007; NMAP, 2008; Siegler et al., 2010). Brown and Quinn (2007a, 2007b) described multiple ways in which algebra critically depends on fraction concepts (e.g., the importance of understanding of slope, a fraction, for linear equations).

Supporting competencies of early math skills

The literature on early math prediction supports contributions of both domain-specific and more general cognitive processes. Domain-specific components typically include magnitude comparison (approximate number system acuity) and/or estimation, and these are potentially foundational to later mathematical outcomes (e.g., Butterworth, 2005; Dehaene & Cohen, 1995; Dehaene, Piazza, Pinel, & Cohen, 2003; Feigenson et al., 2004). These are often collectively referred to as numerosity and are typically distinguished from addition and other arithmetic operations. Mathematics performance also builds on preexisting and developing cognitive processes, including linguistic and spatial skills and especially working memory (e.g., Raghubar, Barnes, & Hecht, 2010). Two widely used models, LeFevre and colleagues (2010) and von Aster and Shalev (2007), emphasize the interaction between domain-general and domain-specific processes for arithmetic but are less clear about their relative contributions to more advanced mathematical skill in the context of more elementary arithmetic. Data bearing on this question often come from the early elementary school grades, when math facts are less likely to be automatized (Ashcraft, 1992; Ashcraft & Christy, 1995; Calhoon, Emerson, Flores, & Houchins, 2007), although it is recognized that this automization is conditional on practice and may vary with age or cultural factors (Geary, Bow-Thomas, Liu, & Siegler, 1996),

Empirical results vary with age and the type of mathematical performance considered, but proficiency in both domain-specific and domain-general areas contribute to outcomes, with the former being more predictive of the earlier developing skills (e.g., basic facts) than later developing or more complex skills (e.g., procedural computation, word problems); in contrast, domain-general processes