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On the distortion of model fit in comparing the bifactor model and the higher-order factor model☆

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ABSTRACT

Gignac (2016) showed that if the constraint by which the higher-order factor model is nested within the bifactor model is violated (the so-called 'proportionality constraint'), model misfit relates strongly to the magnitude of the violation. In the present paper two clarifications of the results by Gignac are discussed. First, it is noted how the misfit relates to the magnitude of the violation in the simulation study by Gignac. Second, it is argued that such a pattern of distortion of model fit as found by Gignac is generally expected and not unique to the bifactor versus higher-order factor model comparison.

Studying the construct of intelligence requires a suitable psychometric model in which intelligence is operationalized in terms of its measures (commonly, the subtest scores of an intelligence test battery). Interestingly, there is yet no consensus with respect to the choice of the psychometric model for intelligence. The discussion seems to focus around two classes of models, non-hierarchical models like the correlated factor model (e.g., Horn, 1968) and hierarchical models like the bifactor model (e.g., Gignac, 2008) and the higher-order factor model (e.g., Johnson, Bouchard, Krueger, McGue, & Gottesman, 2004). The hierarchical models separate general intelligence, or *g* (Jensen, 1998) from specific cognitive abilities like working memory, perceptual organization, and verbal comprehension. The bifactor model and the higher-order factor model differ in the statistical specification by which this distinction is enforced. That is, in the bifactor model, *g* is modeled as an additional cognitive ability underlying all subtests, while in the higher-order factor model, *g* is modeled as a second-order factor underlying the first-order specific cognitive abilities.

Ideally, the decision on which of the models is the most appropriate conceptualization of intelligence is guided primarily by theoretical consideration for which subsequent empirical support is gathered. Concerning the first, no consensus has yet been reached about which of the models has a better theoretical justification. See for instance Horn (1968) for arguments in favor of the correlated factor model, Jensen (1998, p.78) and Johnson and Bouchard (2004) for arguments in favor of the higher-order factor model, and Gignac (2008) and Beaujean (2015) for arguments in favor of the bifactor model.

With respect to the empirical support for the different models, generally, empirical studies seem to focus on the comparison of the bifactor model and the higher-order factor model. These studies tend to show that the bifactor model is better fitting in terms of various goodness-of-fit measures (e.g., Brunner, Nagy, & Wilhelm, 2012; Gignac, 2008; Golay & Lecerf, 2011; Keith, 2005; Watkins & Kush, 2002). However, Murray and Johnson (2013), Morgan, Hodge, Wells, and Watkins (2015) and Maydeu-Olivares and Coffman (2006) pointed out that such fit indices might be biased in favor of the bifactor model. These studies cast doubt over the statistical comparison of the bifactor model and the higher-order factor model.

The statistical difference between the bifactor model and the higher-order factor model is well understood. That is, the higher-order factor model can be seen as a special case of the bifactor model: the higher-order factor model is obtained from a bifactor model by imposing a so-called proportionality constrained on the g-loadings in the bifactor model (Yung, Thissen, & McLeod, 1999). To increase our understanding about the proportionality constraint, Gignac (2016) showed that if the constraint is violated, model misfit is strongly related to the magnitude of the violation. In the present paper two clarifications of the results by Gignac are discussed. First, it is noted how the misfit is related to the magnitude of the violation in the simulation study by Gignac. Second, it is argued that such a pattern of distortion in model fit as found by Gignac is generally expected and not unique to the bifactor versus higher-order factor model comparison.

1. The Gignac study

Gignac (2016) created 12 population correlation matrices according to a model with *g* and 3 specific cognitive abilities. In these matrices,

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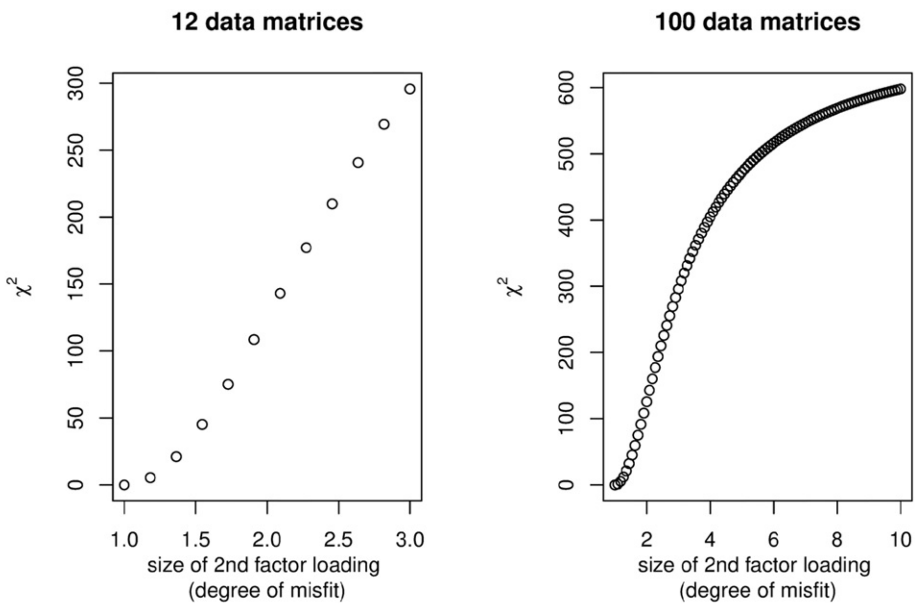

Fig. 1. The χ^2 for the model with an equal factor loading of the first and second subtest based on 12 covariance matrices with the second factor loading increasing from 1 to 3 (left), and based on 100 covariance matrices with the second factor loading increasing from 1 to 10 (right).

violations of the proportionality constraint were introduced with an increasing magnitude such that first correlation matrix did not contain a violation of the proportionality constraint (i.e., the matrix confirms a higher-order factor model) and the twelfth matrix contained a relatively large violation of the proportionality constraint. To these matrices, both the higher-order factor model and the bifactor model were fit and the χ^2, TLI, AIC, and BIC fit indices were determined. In Fig. 2 of the Gignac paper, it is shown that the relation between the difference in these fit measures between the higher-order factor model and the bifactor model tends to be highly linear.

1.1. The relation between magnitude of violation and model fit

As the TLI, AIC, and BIC fit indices are all linear transformations of the χ^2, the four plots in Fig. 2 of Gignac (2016) are identical in essence as they only differ in their mean and variance but not in their configuration (the patterns are exactly identical). Therefore, focus will be only on the χ^2. The χ^2-statistic of a model is obtained by multiplying the so-called 'fit function' with N-1, where N is the number of subjects in the data. Thus for the higher-order factor model:

$$\chi^2_{\text{higher-order}} = (\text{N}-1) \times \text{f}_{\text{higher-order}}(\text{data}|\text{higher-order model parameter estimates}),$$

and for the bifactor model:

$$\chi^2_{\text{bifactor}} = (\text{N}-1) \times \text{f}_{\text{bifactor}}(\text{data}|\text{bifactor model parameter estimates}).$$

As can be seen, the χ^2 depends on N, the data, and the model parameter estimates. If the higher-order factor model and the bifactor model are being compared, one can take the difference in χ^2 of the two models, that is

$$\chi^2_{\text{higher-order vs bifactor}} = \chi^2_{\text{higher-order}} - \chi^2_{\text{bifactor}}$$

This difference -which is referred to as the χ^2 difference test or likelihood ratio test- can then be used to infer whether the model fit of the higher-order factor model significant deteriorates relative to the model fit of the bifactor model.

In the study of Gignac (2016), all the population correlation matrices were generated to confirm to a bifactor model as the correlation matrices contained violations of the higher-order factor model, but no violations of the bifactor model. As population correlation matrices do not include sampling error, the bifactor model fits the correlation matrices perfectly. That is, χ^2_{bifactor} is 0 in all cases (indicating perfect fit). Thus, the difference in χ^2 between the higher-order factor model and the bifactor model equals the χ^2 of the higher-order factor model, that is

$$\chi^2_{\text{higher-order vs bifactor}} = \chi^2_{\text{higher-order}} - \chi^2_{\text{bifactor}} = \chi^2_{\text{higher-order}} - 0 = \chi^2_{\text{higher-order}}$$

This can also be seen from Tables 1, 8, and 9 of the Gignac paper as in these tables the χ^2 values for the bifactor model are all 0. As a result, the χ^2 difference between the higher-order factor model and the bifactor model will depend solely on N and the fit function, that is

$$\chi^2_{\text{higher-order vs bifactor}} = \chi^2_{\text{higher-order}} = (\text{N}-1) \times \text{f}_{\text{higher-order}}(\text{data}|\text{higher-order model parameter estimates}),$$

Two important properties about this difference follow: 1) the difference in χ^2 between the higher-order factor model and the bifactor model correlates perfectly with the sample size (N); and 2) the difference in χ^2 between the higher-order factor model and the bifactor model will be linearly related with the magnitude of the violations of the proportionality constraint only if the fit function is a linear function. However, the fit function is a non-linear function.[1] Therefore, even if the magnitude of the correlations in the data are increased linearly and the model parameters increase linearly (which is not necessarily the case), the difference in χ^2 between the bifactor model and higher-order factor model will not increase linearly due to the nonlinear nature of the fit function.

[1] The fit function is given by: $\text{f}_{\text{higher-order}} = \log(\mathbf{S}) - \log(\mathbf{\Sigma}) + \text{trace}(\mathbf{S}\mathbf{\Sigma}^{-1}) - p$, where $\mathbf{S}$ is the observed covariance matrix, $\mathbf{\Sigma}$ is the model implied covariance matrix evaluated at the parameter estimates, and p is the number of subtests.