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ON NILPOTENT GENERATORS OF THE LIE ALGEBRA \mathfrak{sl}_n

ALISA CHISTOPOLSKAYA

ABSTRACT. Consider the special linear Lie algebra $\mathfrak{sl}_n(\mathbb{K})$ over an infinite field of characteristic different from 2. We prove that for any nonzero nilpotent X there exists a nilpotent Y such that the matrices X and Y generate the Lie algebra $\mathfrak{sl}_n(\mathbb{K})$.

1. INTRODUCTION

It is an important problem to find a minimal generating set of a given algebra. This problem was studied actively for semisimple Lie algebras. In 1951, Kuranishi [5] observed that any semisimple Lie algebra over a field of characteristic zero can be generated by two elements. Twenty-five years later, Ionescu [4] proved that for any nonzero element X of a complex or real simple Lie algebra \mathcal{G} there exists an element Y such that the elements X and Y generate the Lie algebra \mathcal{G} . In the same year, Smith [6] proved that every traceless matrix of order $n \geq 3$ is the commutator of two nilpotent matrices. These results imply that the special linear Lie algebra \mathfrak{sl}_n can be generated by three nilpotent matrices. In 2009, Bois [2] proved that any simple Lie algebra in characteristic different from 2 and 3 can be generated by two elements. He also extended Ionescu's result for classical simple Lie algebras over field of characteristic different from 2 and 3 and proved that graded Cartan type Lie algebra $W(m, \underline{n})$ has this property if and only if $m = 1$. In 2010, Bois [3] showed that for simple graded Lie algebras of Cartan type S , H or K Ionescu's result does not hold.

In this paper we obtain an analogue of Ionescu's result for nilpotent generators of the Lie algebra \mathfrak{sl}_n .

Theorem 1. *Let \mathbb{K} be an infinite field of characteristic different from 2. For any nonzero nilpotent X there exists a nilpotent Y such that the matrices X and Y generate the Lie algebra $\mathfrak{sl}_n(\mathbb{K})$.*

Our interest to this subject was motivated by the study of additive group actions on affine spaces, see [1, Theorem 5.17]. In a forthcoming publication we plan to extend Theorem 1 to arbitrary simple Lie algebras.

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2. MAIN RESULTS

Let \mathbb{K} be an infinite field with $\text{char } \mathbb{K} \neq 2$. A set of elements $\lambda_1, \dots, \lambda_n$ ($\lambda_i \in \mathbb{K}$) is called *consistent* if the following conditions hold:

- (1) $\lambda_1 + \dots + \lambda_n = 0$;
- (2) $\lambda_i \neq 0$ for all i ;
- (3) $\lambda_i \neq \lambda_j$ for all $i \neq j$;

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