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Journal of Economic Theory 163 (2016) 276–287

JOURNAL OF Economic Theory

www.elsevier.com/locate/jet

Optimal robust bilateral trade: Risk neutrality ☆

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Received 11 February 2008; accepted 28 November 2015

Available online 29 December 2015

Abstract

A risk neutral seller and buyer with private information bargain over an indivisible item. We prove that optimal robust bilateral trade mechanisms are payoff equivalent to non-wasteful randomized posted prices.

JEL classification: C78; D40; D82

Keywords: Bilateral trade; Incomplete information; Mechanism design; Robustness; Efficiency

1. Introduction

A seller and a buyer with private information bargaining over an indivisible item is a most fundamental market interaction. It leads to questions of pricing, aggregation of private information and efficiency. Ultimately, what is the set of pricing mechanisms, which are incentive compatible, feasible, robust to the details of traders' information, and, while satisfying these re-

☆ This paper was formerly a part of our paper titled "Ex-Post Constrained-Efficient Bilateral Trade with Risk-Averse Traders" (Copic and Ponsatí, 2008a), the part presented here addresses the case of risk neutrality. We are grateful to Chris Helwig, three anonymous referees and to Emily and Melinda Wang for their typesetting assistance. Čopič is grateful to the Economics Faculty at the University of Ljubljana for their hospitality during Winter 2015.

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http://dx.doi.org/10.1016/j.jet.2015.11.008
0022-0531/

quirements, as efficient as possible (optimal)? A trading mechanism is robust if it satisfies *ex post* incentive compatibility, balanced budget and individual rationality, and it is optimal if it satisfies an appropriate Pareto criterion.[1] Under the assumption that the two traders are risk neutral, we answer this question: such pricing mechanisms are equivalent to non-wasteful randomized posted prices.

A randomized posted price is a common and intuitive mechanism. There is a fixed probability distribution over prices and a price is drawn from that distribution. Each trader then announces whether or not he is willing to trade at that price, which presumably depends on the trader's private valuation of the item. Trade is effected if both traders agree to trade, otherwise no trade takes place. A randomized posted price is non-wasteful if prices at which at least one of the traders would never trade occur with zero probability. Posted prices are commonly used in the real world and albeit in more complex settings, Baye et al. (2006) suggest that price dispersion can at least to some extent be viewed as pure randomization, see also, *e.g.*, Kaplan and Menzio (2015). In the present context, randomized posted prices have been studied by Hagerty and Rogerson (1987), who gave some technical conditions (differentiability of prices and allocations, deterministic mechanisms, and mechanisms, which are step functions), under which robust trading mechanisms are characterized by (non-wasteful) randomized posted prices. As an auxiliary result, we use a different approach to prove their claim in general. This then allows us to apply a suitable notion of optimality and characterize the optimal robust trading mechanisms.[2,3]

It is quite evident that a randomized posted price satisfies *ex post* incentive compatibility, balanced budget and individual rationality, *i.e.*, it is a robust trading mechanism. By misrepresenting his private valuation, for example by saying "no" to trade when the realized price would have made it profitable, a trader can only lose opportunities to make a profit while such misrepresentation brings no potential gains. But it is much less obvious that any pricing mechanism satisfying these properties is equivalent to a randomized posted price. Consider the mechanism whereupon reports $v = (v_1, v_2) \in (0, 1)^2$, the traders trade at a price $p(v)$ with a probability $\varphi(v)$, and with probability $1 - \varphi(v)$ there is no trade,[4] where,

$$p(v) = \begin{cases} \frac{v_1+v_2}{2}, & \text{if } v_1 \le v_2 \\ 0, & \text{otherwise} \end{cases}; \quad \varphi(v) = \begin{cases} \alpha(v_2 - v_1), & \text{if } v_1 \le v_2 \\ 0, & \text{otherwise} \end{cases}; \quad \alpha \in (0, 1]. \tag{1}$$

[1] In their seminal study, Bergemann and Morris (2005) provide a foundation for using *ex post* incentive compatibility as a suitable notion for robustness. In environments with two agents, *ex post* incentive compatibility is equivalent to *interim* implementability on every type space. In private values environments, *ex post* incentive compatibility of a direct revelation mechanism is equivalent to incentive compatibility in dominant strategies. See also Ledyard (1978) and Chung and Ely (2007). A suitable notion of Pareto optimality is the *ex post* constrained optimality, defined by Copic (2015), which is discussed in more detail below.

[2] Other related recent advances in bilateral trade concern weakly undominated strategies, see Yamashita (forthcoming) and Börgers and Smith (2012).

[3] In a very different setting of an exchange economy with a much richer domain of preferences, Barberà and Jackson (1995) prove that dominant-strategy incentive compatible social choice functions are characterized by trading at a finite number of pre-specified proportions, a result that is similar in spirit.

[4] Note that this parametrization is different from the parametrization in, *e.g.*, Myerson and Satterthwaite (1983), where a direct revelation mechanism is parametrized by the probability of trade and the expected transfer. That parametrization was then adopted by Hagerty and Rogerson (1987). When one considers *ex post* individual rationality and budget balance, the price conditional on trade taking place seems more intuitive and suggests a different proof for the general characterization result.