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Analyzing volatility risk and risk premium in option contracts: A new theory☆

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ABSTRACT

We develop a new option pricing framework that tightly integrates with how institutional investors manage options positions. The framework starts with the near-term dynamics of the implied volatility surface and derives no-arbitrage constraints on its current shape. Within this framework, we show that just like option implied volatilities, realized and expected volatilities can also be constructed specific to, and different across, option contracts. Applying the new theory to the S&P 500 index time series and options data, we extract volatility risk and risk premium from the volatility surfaces, and find that the extracted risk premium significantly predicts future stock returns.

1. Introduction

The option pricing literature has made great advances during the past decade; yet large gaps remain between theory and practice. First, traditional option pricing models specify the underlying price and variance rate dynamics and derive their implications on option prices; however, institutional investors manage their volatility views and exchange their quotes not through option prices, but through the option implied volatility computed from the Black-Merton-Scholes (BMS) model. This common practice does not mean that investors agree with the assumptions made by Black and Scholes (1973) and Merton (1973); rather, they use the BMS model as a transformation to enhance quote stability and to highlight the information in the option contract. Second, traditional option pricing theory requires the full specification of the instantaneous

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variance rate dynamics, not only about its current level, but also about its long-run mean; yet in practice, investors do not observe the instantaneous variance rate, but instead observe many option implied volatilities across a wide spectrum of strikes and maturities. Furthermore, investors have much more confidence on how these implied volatilities move in the near term than in the very long run. The map between the implied volatility surface and the instantaneous variance rate dynamics is not always clear or well-determined, forcing modelers to frequently recalibrate their models to match moving market conditions, with each recalibration generating a new set of parameters that are supposed to be fixed over time. Such fudging practices create consistency concerns because the option pricing function would differ if one expects these parameters to be varying over time.

In this paper, we develop a new option pricing framework that tightly integrates with how institutional investors manage their option positions, thus closing the gap between theory and practice. Instead of modeling the full dynamics of an unobservable instantaneous variance rate and deriving the implication on option prices, the new framework models the near-term dynamics of the BMS implied volatility across different strikes and expiries, and derives no-arbitrage constraints directly on the shape of the implied volatility surface. Under the assumed implied volatility dynamics, the shape of the whole implied volatility surface can be cast as the solution to a simple quadratic equation. The computational burden is dramatically reduced compared to the standard option pricing literature. More importantly, by starting with the whole implied volatility surface instead of a single instantaneous variance rate, the new theory does not need to specify the full dynamics, but just the current levels of the drift and the diffusion processes. The current shape of the implied volatility surface only depends on the current levels of its drift and diffusion processes, but does not depend on how these processes will evolve in the future. This "unspanned" nature allows the shape of the current implied volatility surface to be represented as a function of many state variables, but with no fixed model parameters. The high dimensionality renders the model flexible enough to fit the observed implied volatility surface well, whereas the absence of fixed model parameters dramatically simplifies model estimation, alleviates concerns on model stability over time, and allows continuous model recalibration to update the state variables without inducing any intertemporal inconsistency.

The fact that the new theory only specifies the near-term dynamics of the implied volatility surface while leaving its long-term variation unspecified highlights its "semi-parametric" flavor:[1] The theory specifies just enough dynamic structure to achieve a fully parametric characterization of the current implied volatility surface, while saying little about its long-run variation. Traditionally, one can either fit the surface parametrically or nonparametrically. Nonparametric fitting is easy to do, but with concerns that the nonparametrically smoothed implied volatility surface may not satisfy no-arbitrage conditions, may not be extrapolated with stability to regions where data are sparse or unavailable, and the method does not provide a mechanism to reduce the dimension of the surface to a few economically meaningful states. On the other hand, a fully specified parametric model can provide stable and arbitrage-free extrapolation, dimension reduction, and economic interpretation, but it has issues regarding its stability over time, its poor performance when the state dimension is low, and its numerical complexity and instability when the dimension is high. Our semi-parametric theory balances the two by providing a numerically simple approach to readily interpolate and extrapolate the surface while satisfying dynamic no-arbitrage constraints, and to reduce the dimension of the surface to a few economic states while leaving the state dynamics unspecified, thus avoiding introducing any fixed model parameters.

The new theoretical framework does not replace the role played by fully parametric, equilibrium-based option pricing models; instead, it can provide a bridge between market observations and the fundamental valuations from these models. A well-specified parametric option pricing model may not fit the current market observations well, but its valuation can guide future market implied volatility movements. If one believes that option implied volatilities move toward their corresponding fundamental valuations from a parametric model, the new theory can readily embed the fundamental valuations from this model as the near-term targets of the implied volatility movements, and derives no-arbitrage constraints on the current shape of the implied volatility surface with the fundamental valuation as its reference point. To do so, the new theory only asks for the numerical valuation results from the parametric model, without needing to know its parametric model details.

Within the new theoretical framework, we propose a new concept that just like option implied volatilities, both realized and expected volatilities can be made specific to, and different across, option contracts. We define the *option realized volatility* (ORV) at each strike and expiry as the volatility level at which one achieves zero realized profit if one buys the option and performs daily delta-hedge based on the BMS model with this volatility input. Although this realized volatility can be estimated from the realized security price sample path, it is defined against a specific option contract and hence can differ across different strikes and expiries of the reference option contract. Since writing the option at this ORV level generates zero profit, the ex post premium from writing the option at its market price is directly given by the BMS value difference when evaluated at the option's implied volatility and its ORV level, respectively. This new option-specific volatility concept is tightly linked to the common practice of volatility investors, who usually take option positions and perform dynamic delta hedging to separate the volatility exposure from the directional price movement.[2] Taking an option position with delta hedge exposes the investor to

[1] We thank the referee for highlighting this feature.

[2] Indeed, most institutional volatility investors and options market makers are required by their institutions to maintain delta neutrality.