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Two-tier voting: Measuring inequality and specifying the inverse power problem☆

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HIGHLIGHTS

- The coefficient of variation appropriately measures inequality in voting settings.
- The coefficient of variation is appropriate to specify the inverse power problem.
- This specification is equivalent to using a particular distance-based error term.

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ABSTRACT

There are many situations in which different groups make collective decisions by committee voting, with each group represented by a single person. This paper is about two closely related problems. The first is that of how to measure the inequality of a voting system in such a setting. The second is the inverse power problem: the problem of finding voting systems that approximate equal indirect voting power as well as possible. I argue that the coefficient of variation is appropriate to measure the inequality of a voting system and to specify the inverse problem. I then show how specifying the inverse problem with the coefficient of variation compares to using existing objective functions.

1. Introduction

The term two-tier voting refers to situations where different groups have to make a collective decision and do so by voting in an assembly of representatives with one representative per group. Many decisions are taken daily through such voting by all kinds of institutions. The best-studied case is perhaps the Council of the European Union,[1] but it is by far not the only institution making use of some sort of two-tier voting. Other institutions include the UN General Assembly, WTO, OPEC, African Union, German Bundesrat, ECB, and thousands of boards of directors and professional and non-professional associations. The importance of two-tier voting is likely to further increase in the future. Globalization and the emergence of democracy in many parts of the world make collaboration in supra-national organizations more necessary and easier. Furthermore, modern communication technologies facilitate the organization in interest-groups, clubs, and associations, even when the members are geographically dispersed.

The question of how such two-tier voting systems should be designed remains unsolved and certainly cannot be solved in full generality. Nevertheless, there are theoretical concepts that provide guidelines, often stating which voting systems are fair. However, actual voting systems are never completely fair. It is then important to be able to measure how (un)equal a voting system is, i.e. how (un)equal the distribution of influence (or another variable of interest) is that a voting system generates. The inequality measure can then be used to compare voting systems within or across different populations. Such a measure could for example be used to investigate to what extent the inequality of voting systems correlates with other variables, such as income or crime rates. Furthermore, in some cases a voting system that is less equal than another one may have some advantages over

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[1] The literature on two-tier voting within the EU includes, among many others, Baldwin and Widgrén (2004), Beisbart et al. (2005), Felsenthal and Machover (2004), Laruelle and Valenciano (2002), Le Breton et al. (2012), Napel and Widgrén (2006), and Sutter (2000). For an overview of promising (voting) power research avenues see Kurz et al. (2015).

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the more equal one; for example it could be easier to explain its rules to citizens or this voting system could be more easily accepted by the people governed by it. It can then be important to be able to quantify by how much one voting system is more unequal than another one. I suggest to use the coefficient of variation to measure inequality in such voting settings. It can be applied to different variables of interest, such as indirect voting power (as measured by different power measures), the probability that a citizen's preference coincides with the voting outcome, or the number of representatives per citizen in an apportionment context.

Usually, no voting system exists that perfectly implements one of the abstract normative rules on the design of voting systems. The problem of finding voting systems that approximate these theoretical rules is called the inverse (power) problem. To specify the inverse problem, a measure is needed stating how well a voting system corresponds to a theoretical rule. I propose to use the coefficient of variation for this.[2] It turns out that minimizing the coefficient of variation leads to the same outcomes as minimizing the Euclidean distance (of normalized indirect voting power) from the fair ideal. This can be seen as support for the results achieved when using this distance (which cannot be used as an inequality measure in general, i.e. to compare inequality across different populations). This also means that the coefficient of variation has a straightforward interpretation in the context of two-tier voting: it is a transformation of the Euclidean distance to the egalitarian ideal. I furthermore show that using an objective function based on (weighted) voting power at the group level to set up the inverse problem is unsatisfactory. For the discussion of the inverse problem I use a setting where equal indirect Banzhaf power is desired. However, the coefficient of variation can also be applied in a wide variety of other settings (the adaption to other settings is straightforward).

This paper is organized as follows. In Section 2, I describe one of the possible rules prescribing how voting systems should be designed (Penrose's Square Root Rule), which can then be used in the remainder for illustrations. In Section 3, I discuss what properties an inequality measure for voting systems should satisfy and why the coefficient of variation is an appropriate choice. In Section 4, I describe how the inverse power problem can be specified and discuss how this can be done based on the coefficient of variation. In Section 5, I illustrate the use of the coefficient of variation with examples and compare it to using different objective functions. Section 6 concludes.

2. One theoretical concept: Penrose's square root rule

In this section, I introduce one theoretical, abstract rule on how voting systems should be designed, called Penrose's Square Root Rule. I will use this rule as an example in the next sections.[3]

There are N different groups, numbered from 1 to N, each group i consists of n_i individuals, numbered from 1 to n_i. Voting is binary, i.e. a proposal can either be accepted or rejected. Each individual favors the adoption of a proposal with probability one half, independently of all other individuals. Majority voting takes place within each group and the outcome determines the vote of the representative. The representatives of all groups come together in an assembly and it is determined according to their votes in combination with the voting system in the assembly of representatives whether the proposal is adopted or rejected.

Penrose's square root rule: The voting power of (the representative of) a group as measured by the Banzhaf index should be proportional to the square root of its population size.

The main idea of this rule is to make it equally likely for each individual to influence the overall outcome of the two-tier voting procedure, independently of the group she belongs to. If a winning coalition turns into a losing coalition when voter j is excluded we say that voter j has a swing. The absolute Banzhaf index of a voter j is defined as the number of possible winning coalitions that turn into losing coalitions without voter j, divided by the total number of possible coalitions.[4] The normalized or relative Banzhaf index is the absolute Banzhaf index normalized so that the sum of the indices of all voters equals one.

Denote by Ψ_i^B the absolute Banzhaf power index of an individual in group i arising from majority voting in this group and by Φ_i^B the absolute Banzhaf power index of group i in the assembly of representatives, which depends on the voting system in place. Then the probability that an individual in group i has a swing with respect to the overall outcome of the voting procedure (i.e. that she influences with her vote within the group the overall outcome) is Ψ_i^B times Φ_i^B, which is called the indirect Banzhaf voting power. Thus the probability of influencing the overall outcome is equal for all individuals if $\Psi_i^B\Phi_i^B$ is equal for all individuals or equivalently if

$$\Psi_i^B\Phi_i^B = \alpha \tag{1}$$

for some constant $\alpha > 0$ and all i.[5] It can easily be shown that Eq. (1) holds for all i if the normalized Banzhaf index of each group i is equal to

$$\frac{\frac{1}{\Psi_i^B}}{\sum_{j=1}^{N}\frac{1}{\Psi_j^B}}.$$

The normative rule on how to design voting systems as described here states that the indirect voting power $\Psi_i^B\Phi_i^B$ should be equal for all individuals independently of which group they are in, i.e. that Eq. (1) should hold for all i.[6]

3. Measuring the inequality of voting systems

Voting systems in assemblies of representatives are in general not completely fair. Sometimes one may want to quantify how unequal a voting system is. Thus, an inequality measure for a voting system $\mathcal{W}$ in a population consisting of N groups with in total $m = \sum_{i=1}^N n_i$ individuals is needed. I assume that there is a variable of influence or representation at the individual level $r = (r_1, \ldots, r_m)$ with all $r_i \geq 0$ and at least one r_i strictly positive. This variable could for example be indirect Banzhaf power as described in Section 2 so that $r = (\Psi_1^B\Phi_1^B, \ldots, \Psi_1^B\Phi_1^B, \ldots, \Psi_N^B\Phi_N^B, \ldots, \Psi_N^B\Phi_N^B)$.

2 I do not intend to develop algorithms solving the inverse power problem computationally given such a measure, which is what most of the literature does. Finding concrete solutions to the inverse power problem is not trivial; see for example Alon and Edelman (2010), De et al. (2012), Fatima et al. (2008), Kurz (2012), Kurz and Napel (2014), Leech (2003), and De Nijs and Wilmer (2012).

3 I use the most prominent rule on how two-tier voting systems should be designed, but using this rule as illustration does not mean that I endorse it as a normative concept. There are different possible criticisms of this rule, see for example Laruelle and Valenciano (2008). Furthermore, it has been shown that people do not necessarily like voting systems that accord with this rule (Weber, 2015).

4 In the scenario described here, the absolute Banzhaf index of a voter is the probability that this voter has a swing.

5 It is assumed that the grand coalition, i.e. all representatives voting together, can always pass a proposal. This excludes the trivial case $\alpha = 0$.

6 The reason why this is usually referred to as square root rule is the following. Ψ_i^B in Eq. (1) can be approximated by $\sqrt{\frac{2}{\pi n_i}}$, thus Eq. (1) holds if the Banzhaf indices of the groups are proportional to the square root of population size.