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Transient superdiffusion in random walks with a q-exponentially decaying memory profile

Thiago R.S. Moura [a,*], G.M. Viswanathan [a,c], M.A.A. da Silva [b], J.C. Cressoni [b], L.R. da Silva [a,c]

[a] Departamento de Física Teórica e Experimental, Universidade Federal do Rio Grande do Norte, Natal, RN, 59078-900, Brazil

[b] Departamento de Física e Química, FCFRP, Universidade de São Paulo, 14040-903 Ribeirão Preto, SP, Brazil

[c] National Institute of Science and Technology of Complex Systems, Universidade Federal do Rio Grande do Norte, Natal, RN, 59078-900, Brazil

HIGHLIGHTS

- The Fokker–Planck equation is derived with an implicit q-dependence associated with the memory size.
- These results broaden our knowledge on the importance of the diffusive properties of the walker.
- The results shown here pave the way for treating other non-Markovian memory patterns in future work.

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ABSTRACT

We propose a random walk model with q-exponentially decaying memory profile. The q-exponential function is a generalization of the ordinary exponential function. In the limit $q \to 1$, the q-exponential becomes the ordinary exponential function. This model presents a Markovian diffusive regime that is characterized by finite memory correlations. It is well known, that central limit theorems prohibit superdiffusion for Markovian walks with finite variance of step sizes. In this problem we report the outcome of a transient superdiffusion for finite sized walks.

1. Introduction

The random walk model is widely used to describe microscopic transport processes which are characterized by intrinsic randomness. This model and its generalization, the Continuous Time Random Walk (CTRW) set out by E.W. Montroll and G.H. Weiss, are important tools for the study of physical, chemical and economical phenomena [1–3]. One important problem of the random walks is the inclusion of memory correlations in dynamics. The corresponding non-Markovian processes represent a challenge for physicists and mathematicians. The memory effects are incorporated heuristically for physical observables [4] and important for some phenomena, such as turbulence [5] and anomalous diffusion [6]. In organic materials the CTRW is used to describe anomalous transport phenomena [7,8]. In industrial applications, random walk models are important to describe and build new devices, such as amorphous materials used in xerography machines [9] and optical-memory devices [10]. The type of diffusion of a random walker can be determined by studying the asymptotic scaling of the

* Corresponding author.
E-mail addresses: trsmoura@dfte.ufrn.br (T.R.S. Moura), gandhi@dfte.ufrn.br (G.M. Viswanathan), maasilva@fcfrp.usp.br (M.A.A. da Silva), jccressoni@gmail.com (J.C. Cressoni), luciano@dfte.ufrn.br (L.R. da Silva).

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mean square displacement (MSD) with time. The MSD is defined by $\left\langle \Delta x^2 \right\rangle = \left\langle (x - \langle x \rangle)^2 \right\rangle \sim t^{2H}$ where the asymptotic scaling exponent is known as the Hurst exponent [1–3]. The regime is anomalous for $H \neq 1/2$ and normal for $H = 1/2$. For $H < 1/2$ ($H > 1/2$) the regime is termed subdiffusive (superdiffusive). In the model described in the next section we use the Hurst exponent to determine the diffusion regimes of the random walker. We briefly discuss our results in relation to q-Central Limit Theorem (q-CLT) [11,12].

2. The model

We study a model derived from the elephant random walk model (ERW) proposed by G.M. Schütz and S. Trimper [4]. The ERW is termed non-Markovian because the walker keeps a record of the entire history of the walk. The ERW model presents a transition from a normally diffusive scape regime to a superdiffusive regime at $p_c = 3/4$. Some variants of this model have been proposed, such as the (truncated) 'Gaussian random walk' [13], the random walk with exponential memory profile [14] (with exact and numerical solutions leading to an unexpected superdiffusive regime) and the Alzheimer random walk model which exhibits amnestically induced superdiffusion and log-periodic corrections to scaling [15]. The random walk driven by a q-exponential memory is inspired by the recently proposed exponential memory profile model [14].

We use the same notation introduced in the original ERW [4]. The elephant random walk is described as follows: the walker starts at the origin x_0 at time $t_0 = 0$, and at $t > 0$ it moves one step to the left or to the right. The walker keeps a record of the entire history of the walk and the process is described by the stochastic equation

$$x_{t+1} = x_t + \sigma_{t+1}. \tag{1}$$

An equiprobable time t' is chosen from the previous time set $\{1, 2, \ldots, t\}$ at a time $t + 1$. The stochastic variable $\sigma_{t+1} = \pm 1$ is then chosen by the following rule

$$\sigma_{t+1} = \begin{cases} +\sigma_{t'} & \text{with probability} \quad p \\ -\sigma_{t'} & \text{with probability} \quad 1 - p. \end{cases} \tag{2}$$

The first step is always chosen to the right, i.e., $\sigma_1 = +1$. The position at time t thus follows

$$x_t = \sum_{t'=1}^{t} \sigma_{t'} \tag{3}$$

and the second moment is given by Ref. [14]

$$\langle x^2(t) \rangle = \begin{cases} \dfrac{t}{3 - 4p}, & p < \dfrac{3}{4} \\ t \ln t, & p = \dfrac{3}{4} \\ \dfrac{t^{4p-2}}{(3 - 4p)\Gamma(4p - 2)}, & p > \dfrac{3}{4} \end{cases} \tag{4}$$

which are exact relations valid in the asymptotic limit. The diffusion regime is therefore normal for $p < 3/4$ and superdiffusive for $p > 3/4$. For $p = 3/4$ the walk is marginally superdiffusive. The exact propagator is reported to be a Gaussian distribution for all regimes [4], and is described by the following equation

$$P(x, t) = \frac{1}{\sqrt{4\pi D(t)}} \exp\left(-\frac{(x - \langle x(t) \rangle)^2}{4\pi D(t)} \right) \tag{5}$$

with

$$D(t) = \frac{1}{8p - 6} \left[\left(\frac{t}{t_0} \right)^{4p-3} - 1 \right] \tag{6}$$

where $D(t)$ is the diffusion coefficient. It was found recently that the superdiffusive regime is actually characterized by a non-Gaussian distribution [16].

Our model is proposed as a random walk capable to remember past memory events with a memory profile described by the q-exponential probability distribution function. In the ERW model the previous time t' is chosen from a uniform distribution, whereas in this model, a number from the set $\{1, 2, \ldots, t\}$ is randomly chosen from a q-exponential distribution, written as

$$P(t, t') = A \exp_q(-\lambda(t - t')) \tag{7}$$

where

$$\exp_q(-\lambda(t - t')) = [1 + (1 - q)(-\lambda(t - t'))]^{\frac{1}{1-q}}, \tag{8}$$