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Information-theoretic measures for a solitonic profile mass Schrödinger equation with a squared hyperbolic cosecant potential

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HIGHLIGHTS

- Solutions to solitonic profile mass Schrödinger equation with squared hyperbolic cosecant potentials are obtained.
- Such a study might be interesting to those experimental physicists in condensed matter physics.
- The position entropy is calculated considering the singular point.
- The complicated Fourier transforms are derived.
- BBM inequality is verified.

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ABSTRACT

Entropic measures provide analytic tools to help us understand the stability of quantum systems. The spreading of the quantum-mechanical probability cloud for solitonic profile mass Schrödinger equation with a potential $\boldsymbol{V(ax) = -V_0\mathrm{csch}^2(ax)}$ is studied in position and momentum space by means of global (Shannon's information entropy) information-theoretic measures. The position information entropy is considered only for $x > 0$ due to the singular point at $x = 0$. The entropy densities $\rho_s(x)$ and $\rho_s(p)$ are demonstrated and the BBM inequality is saturated.

1. Introduction

It is known that uncertainty relations play an important role in quantum mechanics. Except for the most well-known Heisenberg uncertainty relation [1] in terms of the position and momentum variances, a new uncertainty relations based on the Shannon entropy have been established. Shannon entropy introduced by Claude Shannon in 1948 is used to find fundamental limits on signal processing operations [2]. The original formula is concerned with the discrete variables. In 1970s Bialynicki-Birula and Mycielski obtained a stronger version of the Heisenberg uncertainty principle of quantum mechanics written as $S_x + S_p \geq D(1 + \ln\pi)$, where D represents the spatial dimension [3], the Shannon entropy provides a measure of information about the probability distribution since it is related with the degree of the localization. For this

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uncertainty principle, the position entropy S_x and momentum entropy S_p are defined by

$$S_x = -\int_{-\infty}^{\infty} \rho(x) \ln \rho(x)\, dx \qquad S_p = -\int_{-\infty}^{\infty} \rho(p) \ln \rho(p)\, dp, \tag{1}$$

where $\rho(x) = |\varphi(x)|^2$ and $\rho(p) = |\varphi(p)|^2$ are the probability densities of the particle at the position and at the momentum, respectively. The $\varphi(p)$ represents the wave function in momentum space obtained by Fourier transform. We have recognized that this uncertainty relation has been applied to various quantum systems [4–19]. Also, we have investigated the quantum information entropies for other particular systems [17–20]. It should be pointed out that those quantum systems can be solved analytically, but the calculations of the quantum information entropy can be performed only for a few low-lying states due to the logarithmic factors involved in (1). During the calculation, the key point is the calculation of the Fourier transform for the wave function in position space.

Up to now, we have to point out that the quantum information entropies for those particular quantum systems are investigated within the constant mass case [4–19] except for our recent application to position-dependent mass Schrödinger equation case [20], in which we considered the simplest case, the potential is taken as a null potential, i.e. $V(z) = 0$. The originally complicated differential equation is reduced to a regular constant-mass stationary Schrödinger equation for a particle with the mass m_0. In this work, we attempt to take a more complicated squared hyperbolic cosecant potential $V(ax) = -V_0 \operatorname{csch}^2(ax)$ rather than the null case [20]. We have to emphasize that the hyperbolic potential function was taken as $V(x) = D[1 - \sigma \coth(ax)]^2$ in our recent study [18], in particular whose quantum information entropy was calculated within the *constant mass* Schrödinger equation case. More importantly, the present study is concerned with the existence of the singular point. Such a study has never been treated to our knowledge. The different chosen potentials will result in different solutions and thus the Shannon information entropy is different accordingly. On the other hand, the solutions are expressed as the combinations of the hyperbolic tangent, secant and hypergeometric functions as shown below in Eq. (12). The corresponding Fourier transforms for those low-lying states will be useful and interesting both in mathematics and in physics. Since the present study is concerned with the solitonic profile mass Schrödinger equation with this potential, such a study might be interesting for those experimental physicists from the condensed matter physics through understanding the stability of the quantum systems related to the Shannon entropy.

This paper is organized as follows. In Section 2 we present the exact solutions to this system subject to a squared hyperbolic cosecant potential. In Section 3 we present the normalized wave functions in position space and those in momentum space via Fourier transform. We calculate the position S_x and momentum S_p numerically for low-lying states $n = 0, 1, 2$. The respective probability and entropy densities are illustrated. Finally we give some concluding remarks in Section 4.

2. Solitonic profile mass Schrödinger equation with a squared hyperbolic cosecant potential

For an arbitrary external potential $V(x)$ the position-dependent mass Schrödinger equation ($\hbar = 1$) is written as [21–23]

$$\left\{\nabla_x^2 - m^{-1}(x)\nabla_x m(x)\nabla_x + 2m(x)\left[E - V(x)\right]\right\}\psi(x) = 0, \tag{2}$$

where the mass distribution is taken as $m(x) = m_0(x)\ \operatorname{sech}^2(ax)$, which is a suitable representative of a solitonic profile [24, 25] and has been used widely in condensed matter and low-energy nuclear physics.

As shown in Ref. [23], define $\psi(y) = \cosh^{\tau}(y)\varphi(y)$ and then substitute it into Eq. (2). One has

$$\nabla_y^2\varphi(y) + 2(1+\tau)\tanh(y)\nabla_y\varphi(y) + \left\{\tau(\tau+2)\tanh^2(y) + [\tau + \sigma(E - V(y))]\operatorname{sech}^2(y)\right\}\varphi(y) = 0, \tag{3}$$

where $y = ax$ and $\sigma = 2m_0/a^2$. Consider a new relation $\operatorname{sech}(y) = \cos(z)$ as well as $\tau = -1/2$, then the above equation can be further simplified as

$$-\nabla_z^2\varphi(z) + \tilde{V}(z)\varphi(z) = \varepsilon\varphi(z), \tag{4}$$

where we have defined

$$\tilde{V}(z) = \frac{3}{4}\tan^2(z) + \sigma V(z) + \frac{1}{2}, \qquad \varepsilon = \sigma E. \tag{5}$$

In our recent study [20], the null potential $V(z) = 0$ is taken. That is the simplest physical case. In this work, we consider a special squared hyperbolic cosecant potential $\boldsymbol{V(ax) = -V_0 \operatorname{csch}^2(ax)}$. Certainly, substituting it into Eq. (5) and recalling the relation $\operatorname{sech}(ax) = \cos(z)$, one has $\tilde{V}(z) = 3\tan^2(z)/4 - \mathcal{V}_0\cot^2(z) + 1/2$ with $\mathcal{V}_0 = \sigma V_0$. It is interesting to note that this family of potential represents different potentials in z space as shown in Fig. 1. For example, for $\mathcal{V}_0 > 0$ they look like infinitely deep funnels and behave the potential $1/x$, while for $\mathcal{V}_0 < 0$ they become infinite double-wells and if $\mathcal{V}_0 = 0$ they become the infinite single-well (this case is not illustrated).

For the solution to this system, we take the wave function of the form

$$\varphi(z) = \sin^{\mu}(z)\cos^{\nu}(z)F(z), \tag{6}$$