Physica A 463 (2016) 437–444

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Consensus analysis of switching multi-agent systems with fixed topology and time-delay[☆]

Yongquan Pei, Jitao Sun [*]

Department of Mathematics, Tongji University, Shanghai 200092, China

HIGHLIGHTS

- The multi-agent systems are Markov switching.
- We consider the time-delay in the feedback controller.
- The controller can be calculated by our results.

ARTICLE INFO

Article history:
Received 7 March 2016
Received in revised form 11 May 2016
Available online 29 July 2016

Keywords:
Multi-agent systems
Average consensus
Discrete-time system
Markov switching system
Time-delay
Lyapunov functional

ABSTRACT

This paper investigates the average consensus problems of the discrete-time Markov switching linear multi-agent systems (**LMAS**) with fixed topology and time-delay. Firstly, we introduce a concept of the average consensus to adapt the stochastic systems. Secondly, a time-delay switching consensus protocol is proposed. By developing a new signal mode, the switching signal of the systems and the time-delay signal of the controller can be merged into one signal. Thirdly, by Lyapunov technique, two LMIs criteria of average consensus are provided, and they reveal that the consensus of the multi-agent systems relates to the spectral radius of the Laplacian matrix. Furthermore, by our results and **CCL**-type algorithms, we can get the gain matrices. Finally, a numerical example is given to illustrate the efficiency of our results.

1. Introduction

Distributed multi-agent systems have attracted significant attention in the last decade. The consensus problems of the multi-agent networks are one of the most focused research areas due to its broad applications in many fields including formation control [1], synchronization [2], flocking [3], sensor networks [4] and so on.

The multi-agent systems consist of the agents which can represent robots, humans or animals. Most research of the consensus problems assumes that the dynamics of agent is the controller. For this model, there are many results about continuous-time dynamics [5], discrete-time dynamics [6], first-order dynamics [5], second-order dynamics [7], high-order dynamics [8] and leader-following consensus [9].

In recent years, the linear multi-agent systems (**LMAS**) have attracted several researchers, in which the dynamics of agent is modeled by a linear system. They considered the communication data rate for consensusability [10], the leader-following consensus [11–14], the observer-based protocols [15], the robust consensus control [16] and the event-triggered control [17]. Most works of the consensus problems in the **LMAS** focused on the continuous-time dynamics [12–20]. Results

☆ This work was supported by the National Natural Science Foundation of China (61174039).
* Corresponding author. Fax: +86 2165981985.
E-mail addresses: yqpei0@gmail.com (Y. Pei), sunjt@sh163.net (J. Sun).

http://dx.doi.org/10.1016/j.physa.2016.07.039

about the discrete-time dynamics are less [10,11]. In this paper, we investigate the consensus problems of the discrete-time **LMAS**. The dynamics of each agent is discrete-time linear system.

Switching phenomenon widely exists in the real world. Many papers investigated the consensus problems of the multi-agent systems with switching topology [5,21]. Refs. [11,20,22–25] studied the consensus problems of **LMAS** with switching topology. However, to the best of our knowledge, no one studied the consensus problems of Markov switching **LMAS**. In this paper, the **LMAS** is Markov switching, and the controller is also Markov switching. Adapted for Markov switching **LMAS**, we introduce a concept of the average consensus, which is generalized from the similar concepts in Ref. [26]. We design the controller by the state feedback and the switching signal feedback, and assume they have various delays.

In many papers about consensus problems, it is proved that the consensus of the multi-agent systems relates to the second smallest eigenvalue and the eigenratio (the ratio of the second smallest eigenvalue to the spectral radius) of the Laplacian matrix. Refs. [5,27] presented the convergence rate of the average consensus over an undirected graph is determined by the second smallest eigenvalue of the Laplacian matrix. The eigenratio is an important factor for the consensusability of the multi-agent systems [10]. A larger eigenratio corresponds to better consensusability. By aforementioned results, we can associate with that the spectral radius of the Laplacian matrix is also an important factor for consensus. This is confirmed by our result.

Time-delays are frequently encountered in many practical systems such as engineering, communications and biological systems. For multi-agent systems, time-delay often occurs in information communication. Therefore, communication delay is an inevitable problem, and was considered in many articles [5,7,9,19–21,25]. In this paper, the time-delay is also considered.

Motivated by above, we will solve the consensus problems of Markov switching **LMAS** with fixed topology in this paper. The rest of this paper is organized as follows. Section 2 introduces some graph knowledge and property of Kronecker product. Section 3 presents the consensus problems of discrete-time Markov switching **LMAS** with fixed topology, and defines the average consensus of the stochastic systems. In Section 4, we give two sufficient conditions of consensus, and analyze the relation between consensus and the spectral radius of the Laplacian matrix. Section 5 gives a numerical example to illustrate the efficiency of our results. Concluding remarks are finally stated in Section 6.

Notation: The following notation will be used throughout this paper. **1** (**0**) is a compatible dimension vector with all elements to be one (zero). I_N is the $N \times N$-dimensional identity matrix, and I is the identity matrix of compatible dimensions. The notation $*$ always denotes the symmetric block in one symmetric matrix. The transpose of matrix A is denoted by A^T. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum eigenvalue and the minimum eigenvalue of A respectively. The shorthand $diag\{\cdots\}$ denotes the block diagonal matrix. $\|\cdot\|$ refers to the Euclidean norm for vectors. $E(\cdot)$ stands for the mathematical expectation operator. $\otimes$ denotes the Kronecker product of matrices. Some properties of Kronecker product are useful in this paper: $(A \otimes B)^T = A^T \otimes B^T$, $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$, $A \otimes B + A \otimes C = A \otimes (B + C)$. The Kronecker product of two positive definite matrices is positive definite.

2. Preliminaries

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a *graph* of order N, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_N\}$ is the set of nodes, $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = (a_{ij})_{N\times N}$ is the *weighted adjacency matrix*. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, \ldots, N\}$. $(i, j) \in \mathcal{E}$ denotes there is an edge connect ν_i and ν_j, and ν_i can receive information from ν_j. In the following, it is stipulated that $(i, j) \in \mathcal{E}$ if and only if $a_{ij} > 0$ and $a_{ii} = 0$ for $0 \le i \le N$. If $a_{ij} = a_{ji}$ for all $i, j \in \mathcal{V}$, $\mathcal{G}$ is called an *undirected graph*. If there is a sequence of edges $(i, i_1), (i_1, i_2), \ldots, (i_k, j) \in \mathcal{E}$ for any two agents $i, j \in \mathcal{V}$, $\mathcal{G}$ is called a *connected graph*.

The matrix $\mathcal{L} = (l_{ij})_{N\times N}$ is the *Laplacian matrix* of $\mathcal{G}$, where

$$l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ \displaystyle\sum_{k=1,k\neq i}^{N} a_{ik} & i = j. \end{cases}$$

Lemma 1 ([28]). *Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted undirected graph with the Laplacian $\mathcal{L}$, $\lambda_1 \le \cdots \le \lambda_N$ be the eigenvalues of $\mathcal{L}$. If $\mathcal{G}$ is connected, $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$.*

Assumption 1. In this paper, we assume the communication topology is undirected and connected. Then the eigenvalues of $\mathcal{L}$ are $0 = \lambda_1 < \lambda_2 \le \cdots \le \lambda_N$.

In a multi-agent networks with N agents, the information flow between agents can be described by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. The node ν_i in graph $\mathcal{G}$ corresponds to agent i in the networks. $(i, j) \in \mathcal{E}$ expresses that the information of the agent j can be spread to agent i.

Let $\{r(k), k \in \mathbb{Z}_+\}$ be discrete-time Markov chain, with finite state space $\Upsilon = \{0, 1, \ldots, d - 1\}$. The state transition matrices of $\{r(k)\}$ is $P = (p_{ij})$, where $p_{ij} = \Pr\{r(k + 1) = j | r(k) = i\} \ge 0$, for $i, j \in \Upsilon$, denotes the transition probability from i to j. In this paper, all systems are defined on a complete probability space (Ω, F, P).