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Magnon-bound-state hierarchy for the two-dimensional transverse-field Ising model in the ordered phase

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HIGHLIGHTS

- Spectral properties of the $(2+1)$-dimensional Ising model were investigated.
- Critical amplitude relations for the low-lying levels were calculated.
- Connections with a Z_2 gauge theory conjecture and with possible future experiments were made.
- The numerical diagonalization method was employed.
- Corrections to scaling were suppressed by extended interactions.

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ABSTRACT

In the ordered phase for an Ising ferromagnet, the magnons are attractive to form a series of bound states with the mass gaps, $m_2 < m_3 < \ldots$. Each ratio $m_{2,3,\ldots}/m_1$ (m_1: the single-magnon mass) is expected to be a universal constant in the vicinity of the critical point. In this paper, we devote ourselves to the $(2+1)$-dimensional counterpart, for which the universal hierarchical character remains unclear. We employed the exact diagonalization method, which enables us to calculate the dynamical susceptibility via the continued-fraction expansion. Thereby, we observe a variety of signals including $m_{2,3,4}$, and the spectrum is analyzed with the finite-size-scaling method to estimate the universal mass-gap ratios.

1. Introduction

For an Ising ferromagnet in the ordered phase, the magnons are attractive, forming a series of bound states with the mass gaps, $m_2 < m_3 < \cdots$. As a matter of fact, in $(1+1)$ dimensions, where the system is integrable [1], there exist eight types of elementary excitations with the universal mass gaps

$$
\begin{aligned}
m_2/m_1 &= 2\cos\pi/5 \\
m_3/m_1 &= 2\cos\pi/30 \\
m_4/m_1 &= 4\cos\pi/5\cos 7\pi/30 \\
m_5/m_1 &= 4\cos\pi/5\cos 2\pi/15 \\
m_6/m_1 &= 4\cos\pi/5\cos\pi/30 \\
m_7/m_1 &= 8\cos^2\pi/5\cos 7\pi/30 \\
m_8/m_1 &= 8\cos^2\pi/5\cos 2\pi/15,
\end{aligned}
$$

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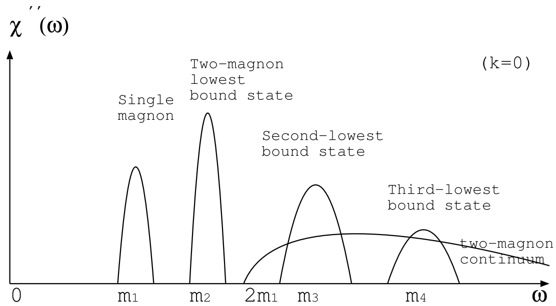

Fig. 1. A schematic drawing of the spectral function for the $(2+1)$-dimensional Ising model in the ordered phase at the zero-momentum $k=0$ sector is shown. The spectrum is expected to exhibit a universal character in the vicinity of the critical point. The m_1 peak corresponds to the single-magnon excitation. The peaks at $m_{2,3,\ldots}$ are the bound states, which may be embedded within the two-magnon continuum extending above $2m_1$.

(m_1: single-magnon mass gap) in the vicinity of the critical point [2,3]. According to the rigorous theory, the *elementary* magnon m_1 is also a composite particle, reflecting a highly non-perturbative character of this problem; in this sense, the underlying physics may lie out of the conventional "magnon" picture. Experimentally, the lowest one $m_2/m_1 = 1.618\ldots$ (golden ratio) was observed [4] for a quasi-one-dimensional quantum Ising ferromagnet, $CoNb_2O_6$ [5,6]. Above $\omega/m_1 \geq 2$, there extends a two-magnon continuum, overwhelming fine details of the spectrum; see Fig. 4 E of Ref. [4], for instance.

In $(2+1)$ dimensions, on the contrary, such rigorous information is not available, and details of the bound-state hierarchy are not fully clarified; the role of dimensionality was argued in §5 of Ref. [7] (see Ref. [8] as well). To the best of author's knowledge, the second-lowest bound state $m_3/m_1 = 2.45(10)$ was detected with the Monte Carlo method [9], whereas the lowest one, $m_2/m_1 \approx 1.81$ [10], has been investigated rather extensively so far [9,11,7,12,13,8]. In this paper, we investigate the $(2+1)$-dimensional Ising model (1) with the exact diagonalization method, which enables us to calculate the dynamical susceptibility via the continued-fraction expansion [14]; note that in the Monte Carlo simulation, one has to resort to the inverse Laplace transform to obtain the spectrum. The spectrum reflects a hierarchical character for $m_{2,3,\ldots}$. In Fig. 1, we present a schematic drawing for a spectral function within the zero-momentum sector.

It has to be mentioned that the magnon-bound-state hierarchy is relevant to the glueball spectrum (screening masses) for the gauge field theory (Svetitsky–Yaffe conjecture) [15–17]. Actually, we show that in the next section, the bound-state hierarchy $m_{2,3,4}$ bears a resemblance to the glueball spectrum for the Z_2-symmetric gauge field theory [16]. Here, we dwell on the characterization of the magnon bound states, and the verification of the conjecture itself lies beyond the scope of this paper.

To be specific, we present the Hamiltonian for the two-dimensional spin-$S=1$ transverse-field Ising model

$$\mathcal{H} = -J\sum_{\langle ij\rangle} S_i^z S_j^z - J'\sum_{\langle\langle ij\rangle\rangle} S_i^z S_j^z - J_4\sum_{\langle ij\rangle}(S_i^z S_j^z)^2 - J_4'\sum_{\langle\langle ij\rangle\rangle}(S_i^z S_j^z)^2 + D\sum_i (S_i^z)^2 - \Gamma\sum_i S_i^x - H\sum_i S_i^z, \quad (1)$$

with the quantum spin-$S=1$ operator $\mathbf{S}_i$ placed at each square-lattice point i. The summations, $\sum_{\langle ij\rangle}$ and $\sum_{\langle\langle ij\rangle\rangle}$, run over all possible nearest-neighbor and next-nearest-neighbor pairs, $\langle ij\rangle$ and $\langle\langle ij\rangle\rangle$, respectively. Correspondingly, J (J_4) and J' (J_4') are the quadratic (biquadratic) interaction parameters. The symbols, D, Γ and H, denote the single-ion anisotropy, the transverse- and longitudinal-magnetic fields, respectively. The parameter D is tunable, and the phase diagram is presented in Fig. 2. Other interaction parameters are set to

$$(J, J', J_4, J_4', \Gamma) = [0.41191697085, 0.16125069616, -0.11764020018, -0.05267926601, 1.0007], \quad (2)$$

so as to suppress corrections to scaling [18]. The (properly scaled) infinitesimal magnetic field $H = 11L^{-y_h}$ [18] with $y_h = 2.481865$ [19] resolves the ground-state two-fold degeneracy [3]. The $S=1$-spin model permits us to incorporate extended interactions so as to suppress corrections to scaling [20]. In fact, as demonstrated in Ref. [18], even for restricted system sizes, the Hamiltonian (1) exhibits suppressed corrections to scaling. Detailed account of the three-dimensional Ising universality is reported in Ref. [19].

The rest of this paper is organized as follows. In Section 2, we present the numerical results. The simulation algorithm is explained as well. In Section 3, we address the summary and discussions.

2. Numerical results

In this section, we present the numerical results for the $(2+1)$-dimensional Ising model (1). We employed the exact diagonalization method for the finite-size cluster with $N \leq 22$ spins. We imposed the screw-boundary condition [21] to