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Stokes efficiency and its stochastic properties

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HIGHLIGHTS

- The notion of the stochastic Stokes efficiency is proposed in a time asymmetric rocked ratchet model.
- The fluctuation in Stokes efficiency dominate its mean value over the larger parameter space of the model.
- The mean Stokes efficiency decreases as we go from adiabatic to nonadiabatic regime.

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ABSTRACT

We study the Stokes efficiency and its fluctuating properties in the case of a spatial asymmetric ratchet potential with a temporal asymmetric driving force from adiabatic to nonadiabatic regime. Our numerical investigations show that the average Stokes efficiency and the average current decrease with increase in the frequency of driving. For low frequency of driving, i.e., in the case of an adiabatic regime, we reproduced the analytical results supporting our numerical simulations. By evaluating the probability distribution $p(\eta_s)$ for Stokes efficiency η_s, we focus on the stochastic properties of Stokes efficiency. We find that in most of the parameter space, fluctuations in η_s are comparable to or larger than the mean values. In such a situation, one has to study the nature of the full probability distribution of η_s. We observe that with increase in frequency of driving, the distribution becomes multipeaked. At the same time, the average Stokes efficiency monotonically decreases with increase in frequency of the drive. For high frequency of driving, the distribution develops a peak at zero. With further increase in frequency of the drive this peak gets sharper. And finally at sufficiently high frequency, we get a strong peak at zero indicating that there is no effective transport in this regime.

1. Introduction

Over the last several decades, the study of interplay of noise and nonlinearity has become a major subject of research in various multidisciplinary areas [1–13]. These studies include the noise induced directed transport, stochastic resonance, noise induced stability of unstable states, noise induced phase transition, noise induced ordering, etc. Also synchronization plays an important role in many processes occurring in nature. The complex interplay of noise and nonlinearity results many interesting phenomena in the behaviour of synchronization in the case of globally coupled phase oscillators [14,15]. In most of the above mentioned cases, the presence of noise is very much essential. In the literature, the noise induced directed transport in periodic extended structures in the absence of overall net bias in the potential has been extensively studied.

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Two basic ingredients are required for the possibility of such directed transport, namely, the system should be driven out of equilibrium and there should be some asymmetry (either temporal or spatial) along with nonlinearity in the system. The systems consisting of Brownian particles and operating with the minimal conditions for such directed transport are known as Brownian ratchets/Brownian motors [16,17]. There is increasing interest from physicists, biologists and engineers on the study of so called ratchets or Brownian motors [18–21]. Based on the various ways of introducing the asymmetry or nonlinearity in the systems, there are various kinds of Brownian ratchet models like flashing ratchets, rocking ratchets, time-asymmetric ratchets and inhomogeneous ratchets [5].

The main motivation in the study of performance characteristics of Brownian ratchet is the notion of efficiency of energy transduction from the thermal fluctuations [22]. With the help of stochastic energetic formalism [23–25] proposed by Sekimoto, efficiency can be defined in a wide class of ratchet models [26,27]. Different kinds of efficiency (thermodynamic, Stoke's and generalized) for ratchet models have been studied in detail both analytically and numerically. Since the Brownian ratchet operates out of equilibrium, there is always an unavoidable heat transfer to the medium/environment, which degrades the efficiency of performance of the ratchet. It has been noticed that Brownian motors based on the principle of flashing ratchet models result in low efficiency as compared to the case of adiabatically changing ratchet potentials, e.g., rocked ratchet models exhibit high value in efficiency [28–32]. In this work, we are mainly interested in the case of spatial asymmetric rocked ratchet models.

In all kinds of ratchet models, it is to be emphasized that the Brownian particle moves in a periodic potential system and hence it ends up with the same potential energy even after crossing over to the adjacent potential minimum. Hence, there is no extra energy stored in the particle which can be used for a given purpose. In order to extract work/energy out of its motion, it is necessary to apply an external load force against which the particle moves and stores energy in the form of potential energy [23,28,33]. In this context, the efficiency of the ratchet can be defined as the output work against the load force per input energy given to the system. This definition of efficiency is called as thermodynamic efficiency and its value is zero when there is no applied load in the ratchet [23]. The key point in the study of Brownian ratchet models is to understand the basic mechanism of operation of molecular motors. These motors move efficiently in a highly viscous medium and in very high noisy environment. The proper measure of efficiency in such viscously loaded molecular motors is known as Stokes efficiency which can be viewed as a measure of how efficiently a motor can utilize the free energy to drive against a viscous drag. Hence, the Stokes efficiency in a ratchet model can be defined as the ratio of the rate of work done against the mean viscous drag and the rate of input energy given to the ratchet [34–36]. The detailed study of Stokes efficiency in case of molecular motors and its comparison with the thermodynamic efficiency is already discussed in Ref. [36]. However, in this work, we are mainly interested in the stochastic properties of Stokes efficiency in a particular rocked ratchet model.

The Brownian ratchets operate at microscopic length scale. For the systems operating at this length scale, the energy exchanged (i.e., in the form of mechanical energy, chemical energy, etc.) between the system and its environment is of the order of thermal energy (few k_BT, k_B being the Boltzmann constant and T being the temperature). Thermal fluctuations play a predominant role, thereby exhibiting distinctly different behaviour from that of macro systems [37]. In the present work, we mainly focus on the study of fluctuating properties of Stokes efficiency in case of a spatial and temporal asymmetric rocked ratchet model. Recently, the enhancement of Stokes efficiency by noise was studied in a microscale domain [38]. In this study, we take interest on the stochastic properties of Stokes efficiency and its probability distribution from adiabatic to nonadiabatic limit. We show that in most of the parameter space, fluctuations dominate the mean values. The mean Stokes efficiency and the average current show a monotonically decreasing behaviour with increase in the frequency of the drive.

2. The model

We consider the dynamics of an overdamped Brownian particle in a periodic spatial asymmetric ratchet potential $V(x)$ in the presence of an external time asymmetric driving force $F(t)$. The motion is governed by the overdamped Langevin equation [27]

$$\gamma\dot{x} = -V'(x) + F(t) + \xi(t), \tag{1}$$

where $x(t)$ denotes the instantaneous position of the particle and γ being the friction coefficient. $\xi(t)$ is the randomly fluctuating Gaussian thermal noise satisfying the properties, $\langle\xi(t)\xi(t')\rangle = (2\,k_BT/\gamma)\delta(t-t')$ and $\langle\xi(t)\rangle = 0$. The angular bracket $\langle\cdots\rangle$ denote the ensemble average over all the realizations of noise. A schematic diagram of the ratchet potential $V(x)$ along with the external drive $F(t)$ is shown in Fig. 1.

$$\begin{aligned} V(x) &= \frac{Q}{\lambda_1}x, \quad x \le \lambda_1 \\ &= \frac{Q}{\lambda_2}(1-x), \quad \lambda_1 < x \le \lambda, \end{aligned} \tag{2}$$

where Q is the height of the ratchet potential. $\lambda = \lambda_1 + \lambda_2$ is the spatial periodicity of the ratchet potential, which is set to unity. $\Delta = \lambda_1 - \lambda_2$ represents the spatial asymmetric parameter which characterizes the asymmetry of the ratchet potential.