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Experimental velocity distributions in a granular submonolayer

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HIGHLIGHTS

- Speed distributions are obtained for granular mixtures of spheres and dimers.
- The results are well-described by a distribution developed for a 1D system.
- At higher packing fractions this distribution is particularly advantageous.
- This distribution may be extended to other mixtures.

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ABSTRACT

Experimental speed distributions are obtained for driven granular submonolayers of binary mixtures of single spheres and dimers of spheres. The results are well-described by a distribution originally developed for a single-species one-dimensional system. This suggests that such a distribution may be extended to other mixtures such as systems exhibiting aggregation and dissociation.

1. Introduction

Granular particles exhibit dissipative collisions [1–5] and negligible Brownian motion and thus, unless energy is injected into the system, lose their kinetic energy [1,3,4]. However, a stationary state with motion can be maintained by injecting energy such as by external agitation [1,3]. The resulting velocity distributions are generally different from those of thermal systems in equilibrium [1,3,4].

In equilibrium, classical systems of thermal particles have a velocity distribution governed by the well-known Maxwell–Boltzmann result:

$$P(\vec{v}) \propto \exp\left(-\frac{M\vec{v}\cdot\vec{v}}{2k_BT}\right), \tag{1}$$

where $P(\vec{v})$ is the probability of a particle having velocity $\vec{v}$, k_BT is the thermal energy and M is the particle mass. Not being in thermodynamic equilibrium, granular particles with energy injection do not generally follow this distribution [1,4].

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However, Puglisi et al. [3] have studied a model of N identical particles on a circle with random, Brownian-like forces, as well as viscous damping, but with dissipative collisions, unlike thermal systems [3]. Dissipative collisions favour clustering [6,4], and this is taken into account by considering the system as consisting of regions ("boxes") containing m particles; each region is assumed to have a Gaussian velocity distribution whose variance, unlike in the thermal case, depends on m. Extrapolating the behaviour of this variance ($\propto m^{-\beta}$) and the distribution of box occupancies (it is governed by a Poisson distribution for the thermal case, but Puglisi et al. [3] found a distribution dependent on a dimensionless parameter α) from simulations, they obtain the following distribution for the stationary state in the limit of large numbers of particles:

$$P(v) \propto \sum_{m=1}^{\infty} \exp\left(-\frac{(v/v_0)^2 m^{\beta}}{2}\right) \exp(-\alpha m), \tag{2}$$

where v is the speed, v_0 is a characteristic speed, and α and β are unitless parameters that can depend on the restitution coefficient. The distribution peaks close to v_0 for small β, and in the limit in which the Maxwell–Boltzmann distribution is recovered, v_0 becomes $\sqrt{k_B T/M}$. Puglisi et al. [3] also report evidence that, at least qualitatively, the results for 2D simulations are comparable.

Experimentally, quasi-2D systems, defined as systems whose constituents cannot have the same 2D coordinates [6–8,5,9,10], can be formed, for instance, by submonolayers of granular spheres. If the model of Puglisi et al. [3] is applicable to two dimensions, then in the x–y plane $\delta N(\vec{v})$, the number of instances of particles with a velocity between $\vec{v}$ and $\vec{v} + \mathrm{d}\vec{v}$, obeys:

$$\delta N(\vec{v}) \propto \sum_{m=1}^{\infty} \exp\left(-\frac{\vec{v}\cdot\vec{v} m^{\beta}}{2\, V_0^2}\right) \exp(-\alpha m) \mathrm{d}v_x \mathrm{d}v_y, \tag{3}$$

where v_x and v_y are the x and y velocity components. In terms of speeds:

$$\delta N(v) \propto \int_{\theta=0}^{\theta=2\pi} \sum_{m=1}^{\infty} \exp\left(-\frac{(v/v_0)^2 m^{\beta}}{2}\right) \exp(-\alpha m) v \mathrm{d}v \mathrm{d}\theta, \tag{4}$$

where the angle θ corresponds to the direction of motion. Assuming the dynamics is on average isotropic, then:

$$\delta N(v) \propto \sum_{m=1}^{\infty} \exp\left(-\frac{(v/v_0)^2 m^{\beta}}{2}\right) \exp(-\alpha m) v \mathrm{d}v, \tag{5}$$

and finally, for finite systems, the sum must be truncated at the number of constituent particles.

Extending this model to mixtures is of obvious practical interest, as they are of great interest in fields such as geophysics [2]. Mixtures of dimers and monomers are an important special case. Dimers and larger aggregates can arise in practice due to capillary or other attractive forces, and dimers or dimer-like particles can also arise due to imperfect processing (such as milling or grinding). It is reasonable to assume the ensuing systems are generally mixtures of monomers, dimers and possibly larger aggregates. In thermal systems, dimers can arise from the aggregation of single particles such as atoms [11], molecules [12,13] or colloids [14,15]. In these instances, the systems normally do not consist exclusively of dimers, but contain both monomers and aggregates instead [11–15]. Furthermore, dimers of spheres can serve as a simple model of non-spherical particles. Mixtures of dimers and monomers can thus serve as important model binary systems.

Research on dynamic granular mixtures has mostly focused on binary mixtures of spheres [16–29], on polydisperse [30–33] and occasionally on tridisperse spheres [26,27], and only a few studies have focused on mixtures of granular particles with different morphologies [34,27]. Therefore, the dynamics of mixtures including non-spherical species, such as mixtures of dimers and monomers, is an important topic of study. Furthermore, Costantini et al. [35] have found that a simulational system of dissipative hard sphere dimers allowed to "cool" freely, i.e. to dissipate all of its kinetic energy, exhibits a Maxwell–Boltzmann velocity distribution. Puglisi et al. [36] have more recently found that for an experimental submonolayer of single spheres with constant, slightly spatially inhomogeneous energy injection the speed distributions are non-Maxwellian, particularly at high speeds. These results suggest that granular systems with energy injection do not follow a Maxwell–Boltzmann distribution whereas those without energy injection do.

Dorbolo et al. [37] have found that monolayers of dumbbells and of trimers consisting of steel spheres forming an equilateral triangle have dynamics substantially different from those of monomers of such spheres. Though they do not report the speed distributions of individual spheres in their composites, it can be reasonably inferred that they are unlikely to have the same functional form as those of the monomers. Importantly, Dorbolo et al. [37] studied particles that are *not* sterically analogous to colliding spheres. In particular, the dumbbells they examined have aspect ratios of 3.5 and of 5.5, whereas two spheres in contact have an aspect ratio of 2. Thus while their work explores the physics of aggregates 1.5 diameters or more further apart than colliding monomers, it cannot be assumed *a priori* that dimers sterically like colliding monomers exhibit the same dynamics.

In the present work, we examine the experimental dynamics of binary mixtures of single spheres and dimers of aspect ratio of 2 with energy injection, and compare our results with the distribution of Puglisi et al. [3].