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A new fuzzy multi-objective higher order moment portfolio selection model for diversified portfolios

Wei Yue [a,b], Yuping Wang [a,*]

[a] School of Computer Science and Technology, Xidian University, Xi'an 710071, China

[b] School of Science, Xi'an Polytechnic University, Xi'an 710071, China

HIGHLIGHTS

- A fuzzy multi-objective higher order moment portfolio selection model is proposed.
- A new entropy function based on Minkowski measure is proposed to obtain better diversification portfolios.
- A new multi-objective evolutionary algorithm is designed to solve the proposed model efficiently.
- Results show the effectiveness of the proposed portfolio mode.

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ABSTRACT

Due to the important effect of the higher order moments to portfolio returns, the aim of this paper is to make use of the third and fourth moments for fuzzy multi-objective portfolio selection model. Firstly, in order to overcome the low diversity of the obtained solution set and lead to corner solutions for the conventional higher moment portfolio selection models, a new entropy function based on Minkowski measure is proposed as a new objective function and a novel fuzzy multi-objective weighted possibilistic higher order moment portfolio model is presented. Secondly, to solve the proposed model efficiently, a new multi-objective evolutionary algorithm is designed. Thirdly, several portfolio performance evaluation techniques are used to evaluate the performance of the portfolio models. Finally, some experiments are conducted by using the data of Shanghai Stock Exchange and the results indicate the efficiency and effectiveness of the proposed model and algorithm.

1. Introduction

Traditional portfolio selection theory is derived from the mean–variance probabilistic model by Markowitz [1], which assumes an underlying normal/quadratic utility distribution for asset's returns. However, a plethora of empirical studies show that the distributions of asset returns usually tend to be of asymmetric leptokurtic and heavy-tailed features, and are not normally distributed [2–4]. This implies that the higher order moments cannot be neglected. Thus the investors should consider higher order moments in their investment decision. Some researchers have tried to study portfolio problems in a three-moment or four-moment framework. For example, Campbell et al. [5] used the mean–variance–skewness framework with the skew normal distribution, and suggested that it is important to incorporate higher order moments in portfolio selection. Adcock [6] studied the mean–variance–skewness portfolio model under the multivariate extended skew-Student

* Corresponding author.
E-mail addresses: yuewei@stu.xidian.edu.cn (W. Yue), ywang@xidian.edu.cn (Y. Wang).

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distribution, and used the quadratic programming to solve the model. Maringer et al. [7] proposed an extension of the classical Markowitz model by taking into consideration the higher order moments and used stochastic algorithms to find the optimal portfolio. Doana et al. [8] studied the systematic skewness and systematic kurtosis of Australian stock returns in the spirit of the higher-moment asset pricing model, and presented the empirical results of examining the roles of systematic skewness and kurtosis. However, these higher order moments portfolio models come with some serious issues, and the major issues are: (a) the error in the estimation of corner solutions, and (b) the low diversity in the portfolio. The low diversity of the portfolio may result in loss while some of the invested assets experience unexpected gains [9,10]. In the last few years, Jana et al. [11] and Usta et al. [12] used the Shannon's entropy to measure the diversification, and generate a well-diversified portfolio. Additionally, Huang [13] used the Shannon's entropy as a constraint condition in the mean–variance framework, and avoided the concentrative portfolio allocation. Recently, Yu et al. [14] discussed the diversified portfolios with different entropy measures, and concluded that the models with Yager's entropy yield the higher economic value of diversification than the portfolio model with Shannon's entropy.

Besides the higher order moments, uncertainty is another important factor in portfolio model because investors may face uncertain, imprecise and vague data. Over the decades, the vast majority of the existing portfolio selection models are based on probability theory under some random states. However, if there is not enough historical data, it is more difficult to use statistical variable to describe model. It is more reasonable to use fuzzy variables [15]. So, in portfolio selection literature, many researchers extended the probabilistic portfolio model to fuzzy environment in different ways. For example, Li et al. [16] developed a fuzzy portfolio selection model with background risk, based on the definitions of the possibilistic theory and used a genetic algorithm to solve the proposed model. Sadatia et al. [17] dealt with a portfolio optimization model involving fuzzy random variables by using the possibility and necessity-based model, and proposed a two-level linear programming model to find the optimum solution. Kocadagli et al. [18] introduced a novel fuzzy portfolio selection model by means of the fuzzy goal programming techniques and presented some numerical examples of a portfolio selection problem to illustrate the effectiveness of the proposed model. More recently, Mashayekhi et al. [19] incorporated the DEA cross-efficiency into Markowitz mean–variance model and proposed a novel fuzzy portfolio model and used NSGA-II algorithm to solve the proposed model.

Also, the transaction cost and the liquidity are very important factor for investors, ignoring transaction costs and the liquidity would result in inefficient portfolios. Some researchers took into account the transaction cost or the liquidity in the portfolio selection model. For instance, Najafi et al. [20] considered the transaction costs, developed a dynamic portfolio selection model and proposed an efficient heuristic method to tackle this problem. Liu et al. [21] discussed the asset allocation in the presence of small proportional transaction costs, which objective is to keep the asset portfolio close to a target portfolio and at the same time to reduce the trading cost in doing so. Oriakhi et al. [22] considered the problem of rebalancing an existing financial portfolio, where transaction costs (fixed and/or changed in nature) have to be paid when the amount of any asset changes. Yu et al. [23] proposed a rebalancing multiple criteria portfolio model by comparing risk, return, skewness, kurtosis, and transaction cost. But they did not consider the portfolio diversification, the liquidity of portfolio and fuzzy return of assets in their model.

It can be seen from above discussion that in order to construct a reasonable portfolio selection model, it is desirable to consider the effect of the following factors: the higher order moments, uncertainty, lower diversity of portfolio, transaction cost and liquidity. However, the existing research works only considered a part of these factors, and few works considered the liquidity of portfolio.

In order to achieve the better portfolio selection and set up a reasonable portfolio selection model, in this paper, in addition to consider the first two order moments (mean and variance) for portfolio selection, we also use the third and fourth order moments (skewness and kurtosis) to design the portfolio selection model. To improve the diversity of portfolio selection and avoid the portfolio concentrating on a few assets, we present a new entropy function based on Minkowski measure, which is defined by the sum of Minkowski distance between weights of the invested assets for the portfolio selection. By using the entropy function, we can directly acquire a well-diversified portfolio. To handle the uncertainty and consider the case with not enough historical data, we introduce the fuzzy variable in the designed model. Furthermore, the transaction cost and liquidity are also considered in our model. Based on these and by making full use of the advantages of three entropies, i.e., the two most commonly used entropies (the Shannon's and Yager's entropies) and our proposed entropy, we construct three different mean–variance–skewness–kurtosis–entropy portfolio models. Moreover, in order to evaluate the performance of these portfolio selection models, we develop a new performance metric based on the Euclidean norms to evaluate the reliability of the solutions. Finally, considering these models are fuzzy multi-objective with liquidity constraints, they become more complicated. Although evolutionary algorithms are an effective way to tackle the multi-objective optimization models and there have been some such algorithms proposed for these problems (e.g. Refs. [24–26]), it is almost impossible to efficiently solve such problems by non-specific-designed algorithms. We design a new efficient multi-objective evolutionary algorithm for these models.

The rest of this paper is organized as follows. Section 2 introduces basic definitions and preliminary results related to fuzzy variables, and give the formulation of the multi-objective optimization problem. In Section 3, we develop a new entropy function based on Minkowski measure to generate a well-diversified portfolio, and present three fuzzy mean–variance–skewness–kurtosis–entropy portfolio models with different entropy functions in the multi-objective framework. Section 4 presents a detailed description of our designed multi-objective evolution algorithm. In Section 5, several portfolio performance evaluation techniques are used to evaluate the performance of these portfolio models, and