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Evolution of a quantum harmonic oscillator coupled to a minimal thermal environment

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HIGHLIGHTS

- Physical system: quantum oscillator (A) coupled to a second oscillator (minimal environment).
- Oscillator A initially in a coherent state and environment in a thermal state.
- Study of the short-time dynamics of the linear entropy of oscillator A.
- Comparison among simple analytical models and the master equation approach.
- Analysis of entropy production and dependence on the effective temperature of the environment.

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ABSTRACT

In this paper it is studied the influence of a minimal thermal environment on the dynamics of a quantum harmonic oscillator (labelled *A*), prepared in a coherent state. The environment itself consists of a second oscillator (labelled *B*), initially in a thermal state. Two types of interaction Hamiltonians are considered, and the time-evolution of the reduced density operator of oscillator *A* is compared to the one obtained from the usual master equation approach, i.e., assuming that oscillator *A* is coupled to a large reservoir. An analysis of the linear entropy evolution of oscillator *A* shows that simplified models may be able to describe important features related to the phenomenon of decoherence, such as the rapid growth of the linear entropy, as well as its dependence on the effective temperature of the environment.

1. Introduction

The coupling of a quantum system to an environment normally leads to the degradation of its non-classical properties. Usually, the environment is modelled by a large number of quantum systems (the reservoir) e.g., a collection of independent harmonic oscillators. However analytical solutions of models involving large reservoirs are virtually impossible to obtain, and approximations are generally necessary. For instance, by assuming a weak system–reservoir coupling, it is possible, via perturbative methods, to derive evolution equations (master equations) for the reduced density operator[1] of the system of interest [1]. This approach, based on the assumption of the existence of a large reservoir, naturally leads to irreversible dynamics of the system variables. Needless to say that such a framework has been particularly useful for the investigation of the quantum to classical transition [2] as well as the phenomenon of decoherence [3,4].

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[1] As one is interested in the evolution of the quantum system itself, a partial trace is taken over the environment variables.

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Nevertheless, the interaction with environments having a small number of degrees of freedom may also cause considerable disturbances to the evolution of quantum systems. An interesting study in this respect is presented in Ref. [5], where it is shown that even a single electron, constituting a "minimal environment" is enough to affect the interference fringes of another electron (system of interest) in an experiment of double photoionization of H_2 molecules. In Ref. [6] it is discussed the behaviour of the specific heat of quantum systems in contact with an environment containing just a single oscillator; the authors conclude that such a simple model is very useful to clarify the occurrence of anomalous effects related to the specific heat of simple systems. In another work [7], it is shown that a very small (but noisy) environment interacting with a bipartite (qubit-oscillator) system may lead to an irreversible-like behaviour. In summary, the above mentioned works show that even minimal environments might be able to cause a considerable degradation of the quantum properties of a system.

The past years have witnessed important developments regarding the manipulation of individual quantum systems, e.g., quantum nanomechanical (or micromechanical) oscillators [8,9]. We may cite, for instance, the cooling of mechanical oscillators to their quantum mechanical ground states [10–12] and the quantum squeezing of motional degrees of freedom in an optomechanical system [13]. Other examples of physical realizations and preparation of states of quantum harmonic oscillators are trapped ions systems [14] and also one mode of the electromagnetic cavity field [15]. A quantum oscillator may be in principle prepared in a variety of quantum states. A pure state that stands out is the coherent state, the "quasiclassical" state of the oscillator defined in the early days of Quantum Theory [16] and a few decades later reintroduced by Klauder [17], Glauber [18] and Sudarshan [19]. Coherent states of the oscillator, here represented by $|\alpha\rangle$,[2] have peculiar statistical properties, e.g., they are minimum uncertainty states in phase space. Besides, they have a characteristic behaviour when in contact with external systems. Namely, if an oscillator initially prepared in a coherent state is assumed to be linearly coupled to a reservoir at $T = 0$ K, its evolution is such that $|\alpha(t)\rangle = |\alpha_0 e^{-\gamma t}\rangle$; here γ is a decay constant related to the oscillator–bath coupling and α_0 is the amplitude of the initial coherent state. Actually, the coherent states are the only pure states that remain pure under dissipation at zero temperature [20]. But as we are going to see, in spite of their robustness at $T = 0$ K, if the oscillator in a coherent state is put in contact with a thermal environment at $T \neq 0$ K, its quantum state evolves to a statistical mixture of pure states i.e., the coherent states are no longer "pointer states". I would like to remark that we often find in the literature discussions about the influence of an environment on superpositions of coherent states (Schrödinger "cat" states) [4,21–26] rather than individual coherent states. Differently from coherent states, though, "cat" states are highly non-classical states [22], and their quantum properties are normally destroyed if they are coupled to an environment even at $T = 0$ K [4,22]. Of course, in a finite temperature environment the situation is even worse [23,24].

Considering the model of reservoir as being a collection of oscillators [1,3,4], the smallest possible environment could be the one consisting of a single oscillator. Thus we would have a system constituted by two coupled quantum harmonic oscillators; oscillator A, the system of interest, and oscillator B, the environment. The problem of two coupled oscillators (e.g., position–position coupling) has been already addressed in the literature; an exact analytical solution (under the rotating wave approximation) was found some time ago [27]. More recently this configuration has been considered for investigating the information transfer between two subsystems [21]. Here I would like to explore the influence of a noisy environment (oscillator B) on the dynamics of the main system (oscillator A). In order to do so, I will consider two distinct forms for the oscillator–oscillator interaction Hamiltonian: position–position (amplitude) and cross-Kerr (phase) couplings. Oscillator A will be assumed to be initially prepared in a pure coherent state $|\alpha_0\rangle_a$, while oscillator B, the minimal environment, will be in a thermal state, a maximally mixed state for a fixed energy. My analysis will be based on the time-evolution of the linear entropy, $\zeta(t) = 1 - Tr\hat{\rho}_A^2(t)$, where $\hat{\rho}_A(t)$ is the reduced density operator of oscillator A, obtained by tracing over the variables of system B, i.e., $\hat{\rho}_A(t) = Tr_B\hat{\rho}(t)$; here $\hat{\rho}(t)$ is the joint density operator. The linear entropy equals zero for a pure state and it is larger than zero for a mixed state. It is therefore a very useful function to quantify the degree of mixture of the quantum state of oscillator A. Evidently in the realm of simple models involving the coupling of an oscillator to a single subsystem, as described above, the evolution of the oscillator will have finite recurrence times, and thus a full irreversible process is not accounted for by those models. Nonetheless, simple analytically solvable models may be useful to gain insights into the general properties of quantum systems. Here, I would like to address the following questions: to what extent simple models (restricted to a sufficiently short time-scale) are able to mimic the decoherence process, compared to a master equation approach? Are they able to appropriately describe the influence of temperature on the dynamics of a quantum oscillator?

This paper is organized as follows: in Section 2 I will present the analytical solutions of the models of system–environment interaction. In Section 3 I will discuss the evolution of the linear entropy of oscillator A. In Section 4 I will summarize the conclusions.

2. Models of environment

2.1. Master equation approach

Firstly I am going to consider the usual (position–position) model of system–environment interaction based on the coupling of a harmonic oscillator to a thermal bath constituted by a collection of uncoupled harmonic oscillators. The

[2] Being α a complex number with $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, for an oscillator associated to creation and annihilation operators $\hat{a}^\dagger$, $\hat{a}$.