Physica A 458 (2016) 266–275

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Steering in spin tomographic probability representation

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HIGHLIGHTS

- Extension of the steering to the case of noncomposite systems.
- Tomographic representation of the correlation function.
- Means of the Correlation Function in the noncomposite systems.

ARTICLE INFO

Article history:
Received 14 October 2015
Received in revised form 14 March 2016
Available online 19 April 2016

Keywords:
Steering
Tomography
Qudit system
Correlation

ABSTRACT

The steering property known for two-qubit state in terms of specific inequalities for the correlation function is translated for the state of qudit with the spin $j = 3/2$. Since most steering detection inequalities are based on the correlation functions we introduce analogs of such functions for the single qudit systems. The tomographic probability representation for the qudit states is applied. The connection between the correlation function in the two-qubit system and the single qudit is presented in an integral form with an intertwining kernel calculated explicitly in tomographic probability terms.

1. Introduction

The problem of the quantum steering was introduced by E. Schrödinger in Ref. [1] as an answer to the paper of A. Einstein, B. Podolsky and N. Rosen [2] to generalize the EPR paradox. Steering reflects the kind of the quantum correlations which exist in the composite quantum systems. There are different kinds of the known quantum correlations as entanglement also available in the composite systems [1]. The quantum correlations available in the noncomposite systems are reflected by the phenomenon of the non contextuality [3]. Since the EPR steering can be applied in one-way quantum cryptography [4,5] or in visualization of the two-qubit state tomography [6] the problem is discussed in a large number of recent papers.

There are many definitions of the term 'steering'. In Ref. [7] the EPR steering was defined as a form of a nonlocality in quantum mechanics, that is in between the entanglement and the Bell nonlocality. In Ref. [8] the notion of steering was reformulated. The EPR steering was considered as the ability of the first system to affect the state of the second system through the choice of the first systems measurement basis. Hence, the concept of the quantum steering can be introduced not only for the multipartite (joint) systems, but also for all systems (including noncomposite ones) with correlations [9]. In Ref. [10] the problem of steering a quantum system from a given state at time t_0 to a new state at time t_1 is discussed.

The EPR steering can be detected through the violation of steering inequalities [5,11–13]. Most of the steering inequalities are connected with the notion of the correlation function [11]. The aim of our paper is to demonstrate that the phenomenon of the quantum steering exists not only in the composite systems but also in the noncomposite systems like the single qudit

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http://dx.doi.org/10.1016/j.physa.2016.04.008

state (e.g. the single qudit with the spin $j = 3/2$ or the four-level atom). Therefore, the main focus of our paper is the correlation function as characteristics of the steering phenomenon for the two-qubit and for the single qudit systems.

The quantum correlations were recently discussed in terms of entropic and information inequalities for von Neumann [14], Tsallis [15] entropies and Umegaki's [16–18] quantum relative entropy. Subadditivity condition and strong subadditivity condition [18,19] are inequalities providing notion of mutual information and conditional information, respectively. They reflect presence of the quantum correlations and the values of these informations characterize the degree of the correlations. It was introduced for bipartite and threepartite systems. Recently, it was shown in Ref. [20] that the latter inequalities are also valid for the single qudit systems. They reflect degree of the quantum correlations between the physical observables related to the single qudits but such that they are analogs of the corresponding observables in the composite systems. Steering also reflects the correlations between the subsystems. In our work we show that analogously to the approach with informations the correlations characterized by steering are available (as steering itself) in indivisible systems.

To clarify the connection of the steering phenomenon in the composite and noncomposite systems we analyze the observables (Hermitian operators [21]) corresponding to the single qudit and the multi qudit systems. We show that there exist in the single qudit system the observables with the properties mathematically identical to the other observables available in the multi qudit system. In this context we map the states of the single qudit systems to the states of the artificial multi qudit systems [22]. To characterize degrees of quantum correlations in the systems we use the tomographic probability representation of quantum mechanics [23]. We find the tomographic representation of the correlation functions that characterize the steering in the quantum system. We introduce the connection between tomograms for the two-qubit system and the tomogram for the single qudit with the spin $j = 3/2$. To introduce the correlation function we consider the specific observable which is a complete analog of the observable used in the two-qubit system but studied in the single qudit with the spin $j = 3/2$ picture. Hence, we can introduce the notion of the steering and detect the steering phenomenon in the system without subsystems. The physical application of the steering phenomena in the single qudit with the spin $j = 3/2$ and for the four-level atom can be performed in the study of the information and the entropic properties of the superconducting multilevel circuits [24,25] where the notion of the artificial two-level atoms playing the role of the qubits is used [26,27].

The paper is organized as follows. In Section 2 we rewrite the correlation function for the two-qubit system in terms of the spin tomogram. The connection of the two partite system tomogram and the single qudit state tomogram is introduced in Section 3. In Section 4 the correlation function is rewritten by means of the tomogram and the notion of the EPR steering is extended to the case of the system without subsystems (a single qudit). The physical meaning of the correlation function in the single qudit systems is given. The prospectives and conclusions are given in Section 5.

2. The two-qubit steering in the spin tomographic representation

The notion of the EPR steering is best studied on the example of the two-qubit state, where each qubit is defined on two dimensional Hilbert space. Let us define the two-qubit quantum system on the Hilbert space. The density matrix of the system state in a four-dimensional Hilbert space $\mathcal{H}_{AB}$ is the matrix ρ_{AB} of the size 4×4 with nonnegative eigenvalues and $\rho_{AB} = \rho_{AB}^{\dagger}$, $Tr\ \rho_{AB} = 1$ hold. The correlations in such system ρ_{AB} can be described by the joint probability function

$$P(a, b|A, B) = \int p_\lambda P(a|A, \lambda)P(b|B, \lambda)\mathrm{d}\lambda, \tag{1}$$

where $P(a|A, \lambda)$ is the probability distribution of the measurement outcomes a under setting A for a hidden variable λ. The hidden variable has the probability distribution p_λ and its hidden state is ρ_λ (a local hidden state (LHS)). If the following model of the correlation

$$P(a, b|A, B) = \int p_\lambda P(a|A, \lambda)Tr(\widehat{\pi}(b|B)\rho_\lambda^{(b)})\mathrm{d}\lambda \tag{2}$$

does not exist, then the state is steerable [11]. Here $\widehat{\pi}(b|B)$ is the projection operator for an observable parameterized by the setting B and the $\rho_\lambda^{(b)}$ is some pure state of the system B. The EPR steering can be detected through the violation of the steering inequalities. The steering inequalities are mostly based on the notion of the correlation function. The quantum correlation function for the two-qubit state is determined by

$$E(\overrightarrow{k_1}, \overrightarrow{k_2}) = Tr(\overrightarrow{k_1} \cdot \overrightarrow{\sigma} \otimes \overrightarrow{k_2} \cdot \overrightarrow{\sigma}\rho), \tag{3}$$

where $\overrightarrow{\sigma}$ is the vector built out of the Pauli matrix, $\overrightarrow{k_1}$, $\overrightarrow{k_2}$ are the unit Bloch vectors of the measurement directions equal to ± 1.

The tomographic probability distribution of the spin states allows to describe the states determined by the density matrix ρ of the two qubits by means of the tomogram. By definition the spin tomogram

$$\omega(\mathbf{x}) = \omega(m_1, m_2, u) = \langle m_1 m_2 | u \rho u^{\dagger} | m_1 m_2 \rangle$$

is the probability to obtain $m_1 = -j_1, -j_1 + 1, \ldots, j_1, m_2 = -j_2, -j_2 + 1, \ldots, j_2, j_{1,2} = 0, 1/2, 1 \ldots$ as the spin projections on directions given by the unitary matrix u. Here we used the notation $\mathbf{x} = (m_1, m_2, u)$ and the matrix u is the unitary