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Can we neglect the multi-layer structure of functional networks?

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HIGHLIGHTS

- The creation of functional representations of multi-layer systems is discussed.
- Single layer functional networks are associated to important oversimplifications.
- Single and multi-layer dynamics on functional networks are compared.

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ABSTRACT

Functional networks, *i.e.* networks representing dynamic relationships between the components of a complex system, have been instrumental for our understanding of, among others, the human brain. Due to limited data availability, the multi-layer nature of numerous functional networks has hitherto been neglected, and nodes are endowed with a single type of links even when multiple relationships coexist at different physical levels. A relevant problem is the assessment of the benefits yielded by studying a multi-layer functional network, against the simplicity guaranteed by the reconstruction and use of the corresponding single layer projection. Here, I tackle this issue by using as a test case, the functional network representing the dynamics of delay propagation through European airports. Neglecting the multi-layer structure of a functional network has dramatic consequences on our understanding of the underlying system, a fact to be taken into account when a projection is the only available information.

1. Introduction

In the early stages of complex network theory [1,2], such paradigm was mainly used to analyze systems whose structure, either physical or virtual, could be directly mapped into a network: examples include transportation systems [3,4], social networks [5] or the World Wide Web [6]. It was soon clear that in certain cases this was not possible, as the only information obtainable from the system itself was the evolution through time of some observables. Such measurable variables reflect the behavior of the interacting elements constituting the system, and as such, the value of every observable is expected to be a *function* of the values of other peers. When the structure of such interactions is inferred from the dynamics of the observables, the result is then called a *functional network*. The introduction of this latter representation has resulted in an important step forward in network science, allowing a broader focus including both structural and dynamical (functional) relations.

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Among the examples of functional network representations, the study of the brain [7,8] and of gene expressions [9,10] are probably the most paradigmatic. In the former, nodes correspond to sensors of a machine recording the activity of the brain. Through the magnetic or electric field generated by spiking neurons, links are then established whenever some kind of synchronization is detected between the recorded time series, usually by means of metrics like Pearson's linear correlation, Synchronization Likelihood [11], or Granger Causality [12]. Similarly, nodes in gene co-expression networks represent individual genes, pairwise connected when some correlation is detected in the dynamics of their respective expression levels.

In the last few years researchers have realized that interactions between the constituting elements of complex systems seldom develop on a single channel. For instance, in a social network, information exchange may happen orally, electronically, or even indirectly. Additionally, people interact according to different types of relationships, *e.g.* friendship and co-working, each one of these affecting the type of information transmitted. Consequently, a correct representation may require different types, or layers, of links [13]. Neglecting such multi-layer structure, or in other words working with the *projected network*, may alter our perception of the topology and dynamics, leading to a wrong understanding of the properties of the system [14,15]. In such cases, therefore, a single-layer network may be an oversimplification, and a multi-layer structure is required.

A single-layer functional representation is generally created by collapsing the dynamics of the individual elements, such that the multiple dynamical aspects of each node are merged into a single time evolution. The same applies to the case of functional brain networks: the six-layer structure of the human cortex is neglected due to the limited spatial resolution of magnetic and electric sensors, and the analyzed time series correspond to the global activity of the top-most layers. Nevertheless, the non-linear nature of the projection process can foster the appearance of constructive or destructive interferences. In other words, a link may appear in the projection even if no relationship is present in any layer; or links in two layers can interfere, and disappear from the projection.

If one has to represent a real system by means of a functional network, and he/she expects it to have a multi-layer structure, it would then be important to understand the magnitude of the distortions created by the use of a single-layer representation. In other words, a fundamental question should be addressed: to what degree single-layer functional networks are representative of the dynamics occurring at different layers?

In order to address this question, here I analyze the European air transportation network and create functional networks modeling the influence airports have on each another, in terms of flight delays and propagation. The availability of high-resolution real data allows the reconstruction of a complete multi-layer picture, in which each layer corresponds to a different airline; dynamics are afterwards collapsed, in order to simulate the creation of a single-layer representation. The resulting structures are compared, both topologically and dynamically, in order to assess whether they provide similar insight into the underlying system.

2. Reconstructing the air transport functional network

The air transportation system can be described both from a physical and a functional perspective. According to the former, the system supports a transport phenomenon, in which passengers and goods are transported between pairs of airports. Therefore, the corresponding network representation encodes the presence of direct connections between nodes, in this case direct flights between airports. Numerous works have focused on this aspect, as for instance Refs. [16,17,4]. A latter perspective disregards the physical movement of items in the network, focusing instead on how the dynamics of nodes is influenced by other peers. Thus, a time evolving observable characterizes each element of the system, its dynamics being a function of the dynamics of its neighboring elements. When such functional relationships are uncovered, the result is a *functional* network. The air transport system can be seen as a collection of airports, whose observables are the delay experienced in each one of them. Clearly, delays are transported by aircraft, and thus the delay observed at one airport is a (potentially noisy) function of other airports delays. Such relationships are then susceptible of being mapped in a functional complex network.

The reconstruction of the physical network is a straightforward process, only requiring an analysis of flights' schedules. On the other hand, the reconstruction of delay functional networks starts with time series representing average landing delays across European airports. In this example, I have considered delay time series for the 50 busiest European airports and the 20 largest airlines in number of flights during 10 consecutive months of year 2011 — see Appendix A for further details. These time series have been pre-processed, in order to ensure their stationarity, for then assessing the synchronization level between pairs of them (see Appendix B).

As depicted in Fig. 1, by starting from the raw time series (bottom left part), two possibilities can be followed. On one side, one can average all of the dynamics corresponding to a node, or in this case to an airport, thus creating a network representing the *projection of the dynamics*. Once one time series per airport is available, different metrics can then be calculated to construct a weighted fully-connected network. On the other side, one can directly create a multi-layer network by considering each airline as a layer, in which afterwards such structure can be projected into a single-layer graph, thus creating a *projection of the topology*.

According to this idea, 23 different networks can be created: following the numeration of Fig. 1, a *projection of the dynamics* (1); two *projections of the topology*, by respectively taking into account the strongest links in each layer (2) and the sum all statistically significant links (3), and the 20 airline networks (4 — 23). All networks, except for number 3, have been pruned in order to have a link density of 0.05.