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On the choice of GARCH parameters for efficient modelling of real stock price dynamics

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HIGHLIGHTS

- Two self-consistent methods to determine GARCH(1,1) parameters are proposed.
- Fitting higher-order moments can lead to inefficient GARCH(1,1) parameter estimation.
- Higher-order moment analysis produces similar results to MLM-based software.
- Fitting Fourier spectrum leads to a more stable GARCH(1,1) stochastic process.
- Fitting spectrum produces a shorter characteristic autocovariance time.

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ABSTRACT

We propose two different methods for optimal choice of GARCH(1,1) parameters for the efficient modelling of stock prices by using a particular return series. Using (as an example) stock return data for Intel Corporation, we vary parameters to fit the average volatility as well as fourth (linked to kurtosis of data) and eighth statistical moments and observe pure convergence of our simulated eighth moment to the stock data. Results indicate that fitting higher-order moments of a return series might not be an optimal approach for choosing GARCH parameters. In contrast, the simulated exponent of the Fourier spectrum decay is much less noisy and can easily fit the corresponding decay of the empirical Fourier spectrum of the used return series of Intel stock, allowing us to efficiently define all GARCH parameters. We compare the estimates of GARCH parameters obtained by fitting price data Fourier spectra with the ones obtained from standard software packages and conclude that the obtained estimates here are deeper in the stability region of parameters. Thus, the proposed method of using Fourier spectra of stock data to estimate GARCH parameters results in a more robust and stable stochastic process but with a shorter characteristic autocovariance time.

1. Introduction

In the 1980s, as the Internet was experiencing a development boom, and electronic forms of communications started replacing paper, trading assets over the wire rapidly gained popularity. Over the Internet, price changes affected by adverse market movement could be updated with unimaginable speed—almost instantly, leading to the development of specialised computer systems, able to process hundreds of trade operations per second [1]. Since then, understanding the risk associated with the trade and acquisition of assets has become of primary importance to traders [2]. Conventional time series operate

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under the assumption of constant variance, while ARCH (*AutoRegressive Conditional Heteroskedasticity*) introduced in 1982 by R. Engle, allows the conditional variance to change over time as a function of past events [3].

Many of today's popular models for risk analysis are based on volatility analysis. The cornerstone of these models is the assumption that a given financial time series is stochastic, and its features must be measured and forecast probabilistically. However, predictable patterns exist. For example, it may often be observed that small changes in the volatility of a financial series tend to be followed by small changes, and large changes tend to foreshadow larger volatility changes [4,5]. Likewise the GARCH (*Generalised ARCH*) model, introduced in 1986 by T. Bollerslev, remains amongst the most well-known models to econometrists. Some of the typical applications of GARCH include, but are not limited to, forecasting stock market volatility [6], predicting day-ahead electricity prices [7], predicting value at risk [8], predicting volatility in foreign exchange rates [9], forecasting crude oil price [10], and stock return correlation analysis [11].

Along with various modifications to the GARCH model, different ways of constructing GARCH have been developed. These aim to enhance the GARCH family by, for example, introducing other, non-Gaussian probability density functions, such as Lévy flight [12], student's t-distribution [13], and exponential [14]. The first two distributions are useful for their 'heavy-tailed' properties, which commonly resemble those found in stock return distributions. The exponential distribution has a use in its own EGARCH (*Exponential GARCH*) model, which is particularly useful due to its ability to account for the 'leverage effect' of a stock price series. Nevertheless, a consistent choice of parameters for GARCH simulations is still an open question. In general this choice can depend on a particular simulation problem and modelling targets. However, we noticed that the desirable stochastic properties of GARCH (for instance, higher-order moments of distribution) can hardly be reachable due to their slow convergence, so alternative statistics properties have to be used to determine simulation parameters.

In Section 2 we introduce the GARCH model and discuss the problem of a self-consistent choice of the model parameters. In Section 3 we discuss two different approaches to determine all three parameters of the model and demonstrate a very slow convergence of GARCH simulations to fit empirical higher-order moments of distribution, using (as an example) a stock return series for Intel. We also demonstrate a much better convergence of the Fourier spectra for the simulated data. In Section 4 we compare our estimates derived both by using stock return Fourier spectra and higher-order kurtosis values for several stocks to the estimates obtained by employing one of the standard available packages. The main conclusion is that the GARCH parameters estimated by simulating the standard package occur very close to the boundary separating stable and unstable stochastic process.

Our proposed method of stock return Fourier spectra results in a more stable and robust GARCH process but with a shorter autocovariance characteristic time. In contrast, our proposed method involving higher-order kurtosis data provides a set of parameters which are quite similar to the standard method (even closer to the stability boundary with even a larger autocovariance characteristic time).

2. The GARCH model

One of the goals of the GARCH model is to better describe the dynamics of conditional variance of a financial return series. GARCH not only links predicted variance to the past events of the process itself, but also introduces dependence of the process on the historical variance of the series [2]. This property allows GARCH to better capture price dynamics as opposed to ARCH. In comparison, ARCH requires more parameters for accurate modelling of a price series, but as the number of parameters to be estimated grows, computation becomes more demanding, and estimation of those parameters burdensome. This is why GARCH replaced ARCH in popularity for the numerous future extensions to the model [2].

GARCH(p,q) is defined by [15]

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{1}$$

where $\alpha_0, \alpha_1, \ldots, \alpha_q$ and $\beta_1, \ldots, \beta_q$ are positive parameters, x_t is a random variable with 0 mean, and σ_{t-j}^2 is the conditional fluctuating variance of the process at a previous time step, $t-j$. As with ARCH, σ_t^2 is the conditional variance output of the process, characterised by a conditional probability density function, $f_t(x)$, often chosen to be Gaussian. Despite the fact that the conditional variance σ_t^2 fluctuates, the process has unconditional variance $\sigma^2 = \langle \sigma_t^2 \rangle = \alpha_0/(1 - \sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j)$ and higher moments $\langle x_t^{2m} \rangle$ (with integer m), if the GARCH parameters satisfy certain constrains.

The first-order GARCH(1,1) model is defined as

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{2}$$

The analytical variance of GARCH(1,1) is given by [15]

$$\sigma^2(\alpha_0, \alpha_1, \beta_1) = \frac{\alpha_0}{1 - \alpha_1 - \beta_1} \tag{3}$$

and the analytical kurtosis can be written as

$$k(\alpha_1, \beta_1) = 3 + \frac{6\alpha_1^2}{1 - 3\alpha_1^2 - 2\alpha_1\beta_1 - \beta_1^2}. \tag{4}$$