



# Discreteness induced extinction



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## HIGHLIGHTS

- It presents an original perspective on the problem.
- Presents an example of how wrong a description based on mean-field theory can be.
- Poses new questions that may be of interest to the scientific community.

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## ABSTRACT

Two simple models based on ecological problems are discussed from the point of view of non-equilibrium statistical mechanics. It is shown how discreteness may be the results of the models that include spatial distribution with discrete interactions when compared with the continuous analogous models. In the continuous case we have, under certain circumstances, the population explosion. When we take into account the finiteness of the population, we get the opposite result, extinction. We will analyze how these results depend on the dimension  $d$  of the space and describe the phenomenon of the “Discreteness Inducing Extinction” (DIE). The results are interpreted in the context of the “paradox of sex”, an old problem of evolutionary biology.

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## 1. Introduction

Statistical mechanics focuses on the analysis of systems that include a large number of simple components. Regardless of the particular nature of these fundamental entities, it describes the interactions among them and the global properties that appear at a macroscopic scale. These emergent phenomena are the hallmark of complex systems. Such systems are used to model processes in many disciplines, most of the times, far from the physical sciences such as biology [1,2], ecology [3–7], economics [8–11] or sociology [12,13]. It provides a scenario that makes possible to encapsulate the huge number of microscopic degrees of freedom of a complex system into just a few collective variables.

The need to take into account aspects of complexity that are routinely neglected is widely recognized [14]. One reason that an answer to some problems in biology [15,16] or in other sciences [17–20] are so difficult to obtain is that many mathematical models have focused on populations that are infinite in size, unstructured, and isolated from other species [21,22]. Satisfactory explanations should consider *finite* populations of agents that interact in an environment where *structure* and *complexity* are able to *emerge* [6,23].

To incorporate a few of these elements in the context of population dynamics of asexual and sexual species treated here, we take into account the *discrete* nature of the individuals interactions [23,24]. A general form of harmful interaction

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between individuals with its pathogens/parasites/predators (ppp's) is considered in a  $d$ -dimensional space. Discreteness of the interactions is a consequence of the *finiteness* of the populations, interactions with ppp's characterize the *non*-insulation explicitly, and a  $d$ -dimensional space enables *structure* and *complexity to emerge*.

We know that if we want information about the emergent aggregate macroscopic behavior of complex systems, we will need to consider the corpuscular character of interacting species [24].<sup>1</sup> We will achieve this goal by employing dynamic renormalization group (DRG) theory to obtain the renormalization group (RG) flow in the parameter space [25,26]. Starting from the microscopic formulation of the model described by reactions, this RG flow will allow us to understand *how the model parameters scale in space and time*. In turn, this information will be helpful in determining the final equilibrium state of the aggregates of interacting species. Such aggregates imply the advantage at the group-level, what appears to be part of the explanation for the existence of altruism in individuals in social species [27–30]. This approach is consistent with the perspective of extensive time and space scales taken by macro-ecology and biogeography [31–34]. Large-scale spatial correlations are likely to be important for the understanding of the evolutionary influence of predator and prey or host and pathogen ecology [35].

The article is organized as follows: Section 2 motivates and describes the models. Section 3 includes the results with subsequent discussion in Section 4. The conclusion is in Section 5 and the last section is an [Appendix](#) with a brief description of the technique of dynamic renormalization group employed as well some pertinent calculations.

## 2. Models

In this subsection we present the two models used. They are simplified models that attempt to capture only the essential aspects of population dynamics. The first is motivated by the competition between an asexual species and a ppp that can harm it, eventually inducing death. The second is the analogous model for the sexual species. The models are as follows.

### Asexual model

The model for asexual species is described by the following reactions, occurring in a  $d$ -dimensional lattice:



The first reaction on the left describes the reproduction of species  $A$  which occurs at rate  $\lambda$  per time unit, at a given site of the lattice. The second reaction describes the attack that species  $A$  suffers from its ppp  $B$ . In this attack, species  $A$  will be annihilated at a rate of  $\mu$  per time unit per population size unit. Note that in a model that takes the spatial character of the interactions in a  $d$  dimensional lattice, both species move diffusely, but the species  $B$  are not created or annihilated. We are interested in the reproductive dynamics of species  $A$  and not in the dynamics of competition of the species  $B$ . Even if it exists, nothing will change the fact that there will always be an inducer of death for species  $A$ . In other words, we are considering two minimum models where only two fundamental aspects of life are included: reproduction and death. No matter what the agents that reproduce do, there will always be something inducing his death. This model captures the essence of the ppp's *being everywhere* [36].

### Sexual model

The model for sexual species is described by the following reactions:



The first reaction describes species  $A$ 's reproduction that occurs at rate  $\lambda' (< \lambda)$  per time unit per population size unit, always at a given site of the lattice. Because two agents are required to reproduce a third, this reaction captures the cost of finding a mate and thus includes the associated Allee effect. The inequality  $\lambda' < \lambda$  is justified by the fact that it is necessary a male and a female for reproduction in no hermaphrodite (diecious) species. Consider a female in a diecious population with a one to one ratio of females to males. For each encounter with another individual, the female encounters a male with probability 1/2. In a hermaphrodite population, every encounter of a hermaphrodite would be with a compatible mating partner. All else being equal (encounter rate, number of off-spring produced per mating pair), the hermaphrodite population can reproduce at twice the rate of the diecious population. For our purposes, this difference is irrelevant (see results below) and in the equations below there will be no distinction between  $\lambda'$  and  $\lambda$ . Everything else is as in the previous model.

This is not the first time that a model of this type is proposed for population dynamics incorporating the *Allee effect* [37–39] on the lattice. For recent references, see Refs. [40,41]. In the process known as the quadratic contact process (QCP) [42], we also have similar reactions. QCP is sometimes called the process of sexual reproduction [43].

<sup>1</sup> As we are interested in population dynamics that occurs in very large distance scales, we can treat individuals of the species as interacting particles.

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